

## Formulario A: Integrales

En este formulario:  $a, b, p, q, C \in \mathbb{R}$  son constantes reales,  $m, n \in \mathbb{N}$  son enteros positivos y  $u = u(x)$  y  $v = v(x)$  son funciones que dependen  $x$ .

### Fórmulas básicas

1.  $\int 0 dx = C$
2.  $\int k dx = kx + C$
3.  $\int (a \cdot u \pm b \cdot v) dx = a \int u dx \pm b \int v dx + C$
4.  $\int u^n du = \frac{u^{n+1}}{n+1} + C; \forall n \neq -1$  regla de la potencia
5.  $\int u dv = uv - \int v du$  integración por partes
6.  $\int a^u du = \frac{a^u}{\ln(a)} + C$
7.  $\int \frac{du}{u} = \ln|u| + C$
8.  $\int e^u dx = e^u + C$

### Fórmulas trigonométricas

9.  $\int \sin(u) du = -\cos(u) + C$
10.  $\int \cos(u) du = \sin(u) + C$
11.  $\int \tan(u) du = \begin{cases} \ln[\sin(u)] + C \\ -\ln[\cos(u)] + C \end{cases}$
12.  $\int \cot(u) du = \ln[\sin(u)] + C$
13.  $\int \sec(u) du = \begin{cases} \ln[\sec(u) + \tan(u)] + C \\ \ln\left[\tan\left(\frac{u}{2} + \frac{\pi}{4}\right)\right] + C \end{cases}$
14.  $\int \csc(u) du = \begin{cases} \ln[\csc(u) - \cot(u)] + C \\ \ln\left[\tan\left(\frac{u}{2}\right)\right] + C \end{cases}$
15.  $\int \sec^2(u) du = \tan(u) + C$
16.  $\int \csc^2(u) du = -\cot(u) + C$
17.  $\int \tan^2(u) du = \tan(u) - u + C$
18.  $\int \cot^2(u) du = -\cot(u) - u + C$
19.  $\int \sen^2(u) du = \begin{cases} \frac{u}{2} - \frac{\sen(2u)}{4} + C \\ \frac{1}{2}[u - \sen(u)\cos(u)] + C \end{cases}$

$$20. \int \cos^2(u) du = \begin{cases} \frac{u}{2} + \frac{\sen(2u)}{4} + C \\ \frac{1}{2}[u + \sen(u)\cos(u)] + C \end{cases}$$

$$21. \int \sec(u)\tan(u) du = \sec(u) + C$$

$$22. \int \csc(u)\cot(u) du = -\csc(u) + C$$

### Fórmulas trigonométricas hiperbólicas

23.  $\int \senh(u) du = \cosh(u) + C$
24.  $\int \cosh(u) du = \senh(u) + C$
25.  $\int \tanh(u) du = \ln[\cosh(u)] + C$
26.  $\int \coth(u) du = \ln[\senh(u)] + C$
27.  $\int \sech(u) du = \begin{cases} \sen^{-1}[\tanh(u)] + C \\ 2\tanh^{-1}(e^u) + C \end{cases}$
28.  $\int \csch(u) du = \begin{cases} \ln\left[\tanh\left(\frac{u}{2}\right)\right] + C \\ -2\coth^{-1}(e^u) + C \end{cases}$
29.  $\int \sech^2(u) du = \tanh(u) + C$
30.  $\int \csch^2(u) du = -\coth(u) + C$
31.  $\int \tanh^2(u) du = u - \tanh(u) + C$
32.  $\int \coth^2(u) du = u - \coth(u) + C$
33.  $\int \senh^2(u) du = \begin{cases} \frac{\senh(2u)}{4} - \frac{u}{2} + C \\ \frac{1}{2}[\senh(u)\cosh(u) - u] + C \end{cases}$
34.  $\int \cosh^2(u) du = \begin{cases} \frac{\senh(2u)}{4} + \frac{u}{2} + C \\ \frac{1}{2}[\senh(u)\cosh(u) + u] + C \end{cases}$
35.  $\int \sech(u)\tanh(u) du = -\sech(u) + C$
36.  $\int \csch(u)\coth(u) du = -\csch(u) + C$

## Fórmulas con $au + b$

37.  $\int \frac{du}{au+b} = \frac{1}{a} \ln(au+b) + C$

38.  $\int \frac{u}{au+b} du = \frac{u}{a} - \frac{b}{a^2} \ln(au+b) + C$

39.  $\int \frac{u^2}{au+b} du = \frac{(au+b)^2}{2a^3} - \frac{2b(au+b)}{a^3} + \frac{b^2}{a^3} \ln(au+b) + C$

40.  $\int \frac{u^3}{au+b} du = \frac{(au+b)^3}{3a^4} - \frac{3b(au+b)^2}{2a^4} + \frac{3b^2(au+b)}{a^4} - \frac{b^3}{a^4}$

$\ln(au+b) + C$

41.  $\int \frac{du}{u(au+b)} = \frac{1}{b} \ln\left(\frac{u}{au+b}\right) + C$

42.  $\int \frac{du}{u^2(au+b)} = -\frac{1}{bu} + \frac{a}{b^2} \ln\left(\frac{au+b}{u}\right) + C$

43.  $\int \frac{du}{(au+b)^2} = -\frac{1}{a(au+b)} + C$

44.  $\int \frac{u}{(au+b)^2} du = \frac{b}{a^2(au+b)} + \frac{1}{a^2} \ln(au+b) + C$

45.  $\int \frac{u^2}{(au+b)^2} du = \frac{au+b}{a^3} - \frac{b^2}{a^3(au+b)} - \frac{2b}{a^3} \ln(au+b) + C$

46.  $\int \frac{du}{u(au+b)^2} = \frac{1}{b(au+b)} + \frac{1}{b^2} \ln\left(\frac{u}{au+b}\right) + C$

47.  $\int \frac{du}{u^2(au+b)^2} = -\frac{a}{b^2(au+b)} - \frac{1}{b^2 u} + \frac{2a}{b^3} \ln\left(\frac{au+b}{u}\right) + C$

48.  $\int \frac{du}{(au+b)^3} = -\frac{1}{2(au+b)^2} + C$

49.  $\int \frac{u}{(au+b)^3} du = -\frac{1}{a^2(au+b)} + \frac{b}{2a^2(au+b)^2} + C$

50.  $\int \frac{u^2}{(au+b)^3} du = \frac{2b}{a^3(au+b)} - \frac{b^2}{2a^3(au+b)^2} + \frac{1}{a^3} \ln(au+b) + C$

51.  $\int (au+b) du = \frac{(au+b)^2}{2a} + C$

52.  $\int (au+b)^n du = \frac{(au+b)^{n+1}}{(n+1)a} + C \quad \forall n \neq -1$

53.  $\int u(au+b)^n du = \frac{(au+b)^{n+2}}{(n+2)a^2} - \frac{b(au+b)^{n+1}}{(n+1)a^2} \quad \forall n \neq -1, -2$

54.  $\int u^2(au+b)^n du = \frac{(au+b)^{n+3}}{(n+3)a^3} - \frac{2b(au+b)^{n+2}}{(n+2)a^3} +$

$\frac{b^2(au+b)^{n+1}}{(n+1)a^3} + C \quad \forall n \neq -1, -2, -3$

55.  $\int u^m(au+b)^n du = \begin{cases} \frac{u^{m+1}(au+b)^n}{m+n+1} + \frac{nb}{m+n+1} \int u^m(au+b)^{n-1} du \\ \frac{u^m(au+b)^{n+1}}{(m+n+1)a} - \frac{mb}{(m+n+1)a} \int u^{m-1}(au+b)^n du \\ -\frac{u^{m+1}(au+b)^{n+1}}{(n+1)b} + \frac{m+n+2}{(n+1)b} \int u^m(au+b)^{n+1} du \end{cases}$

## Fórmulas con $\sqrt{au+b}$

56.  $\int \frac{du}{\sqrt{au+b}} = \frac{2\sqrt{au+b}}{a} + C$

57.  $\int \frac{u}{\sqrt{au+b}} du = \frac{2(au-2b)}{3a^2} \sqrt{au+b} + C$

58.  $\int \frac{u^2}{\sqrt{au+b}} du = \frac{2(3a^2u^2 - 4abu + 8b^2)}{15a^3} \sqrt{au+b} + C$

59.  $\int \frac{du}{u\sqrt{au+b}} = \begin{cases} \frac{1}{\sqrt{b}} \ln\left(\frac{\sqrt{au+b} - \sqrt{b}}{\sqrt{au+b} + \sqrt{b}}\right) + C \\ \frac{2}{\sqrt{-b}} \tan^{-1}\sqrt{\frac{au+b}{-b}} + C \end{cases}$

60.  $\int \frac{du}{u^2\sqrt{au+b}} = -\frac{\sqrt{au+b}}{bu} - \frac{a}{2b} \int \frac{du}{u\sqrt{au+b}}$

61.  $\int \sqrt{au+b} du = \frac{2\sqrt{(au+b)^3}}{3a} + C$

62.  $\int u\sqrt{au+b} du = \frac{2(3au-2b)}{15a^2} \sqrt{(au+b)^3} + C$

63.  $\int u^2\sqrt{au+b} du = \frac{2(15a^2u^2 - 12abu + 8b^2)}{105a^3} \sqrt{(au+b)^3} + C$

64.  $\int \frac{\sqrt{au+b}}{u} du = 2\sqrt{au+b} + b \int \frac{du}{u\sqrt{au+b}}$

65.  $\int \frac{\sqrt{au+b}}{u^2} du = -\frac{\sqrt{au+b}}{u} + \frac{a}{2} \int \frac{du}{u\sqrt{au+b}}$

66.  $\int \frac{u^m}{\sqrt{au+b}} du = \frac{2u^m\sqrt{au+b}}{(2m+1)a} - \frac{2mb}{(2m+1)a} \int \frac{u^{m-1}}{\sqrt{au+b}} du$

67.  $\int \frac{du}{u^m\sqrt{au+b}} = -\frac{\sqrt{au+b}}{(m-1)bu^{m-1}} - \frac{(2m-3)a}{(2m-2)b} \int \frac{du}{u^{m-1}\sqrt{au+b}}$

68.  $\int u^m\sqrt{au+b} du = \frac{2u^m}{(2m+3)a} (au+b)^{\frac{3}{2}} - \frac{2mb}{(2m+3)a} \int u^{m-1}\sqrt{au+b} du$

69.  $\int \frac{\sqrt{au+b}}{u^m} du = -\frac{\sqrt{au+b}}{(m-1)u^{m-1}} + \frac{a}{2(m-1)} \int \frac{du}{u^{m-1}\sqrt{au+b}}$

70.  $\int \frac{\sqrt{au+b}}{u^m} du = -\frac{(au+b)^{\frac{3}{2}}}{(m-1)bu^{m-1}} - \frac{(2m-5)a}{(2m-2)b} \int \frac{\sqrt{au+b}}{u^{m-1}} du$

71.  $\int (au+b)^{\frac{m}{2}} du = \frac{2(au+b)^{\frac{m+2}{2}}}{a(m+2)} + C$

72.  $\int u(au+b)^{\frac{m}{2}} du = \frac{2(au+b)^{\frac{m+4}{2}}}{a^2(m+4)} - \frac{2b(au+b)^{\frac{m+2}{2}}}{a^2(m+2)} + C$

73.  $\int u^2(au+b)^{\frac{m}{2}} du = \frac{2(au+b)^{\frac{m+6}{2}}}{a^3(m+6)} - \frac{4b(au+b)^{\frac{m+4}{2}}}{a^3(m+4)} +$

$\frac{2b^2(au+b)^{\frac{m+2}{2}}}{a^3(m+2)} + C$

74.  $\int \frac{(au+b)^{\frac{m}{2}}}{u} du = \frac{2(au+b)^{\frac{m}{2}}}{m} + b \int \frac{(au+b)^{\frac{m-2}{2}}}{u} du$

75.  $\int \frac{(au+b)^{\frac{m}{2}}}{u^2} du = -\frac{(au+b)^{\frac{m+2}{2}}}{bu} + \frac{am}{2b} \int \frac{(au+b)^{\frac{m}{2}}}{u} du$

76.  $\int \frac{du}{u(au+b)^{\frac{m}{2}}} = \frac{2}{b(m-2)(au+b)^{\frac{m-2}{2}}} + \frac{1}{b} \int \frac{du}{u(au+b)^{\frac{m-2}{2}}}$

### Fórmulas con $u^2 + a^2$

77.  $\int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$

78.  $\int \frac{u}{u^2 + a^2} du = \frac{1}{2} \ln(u^2 + a^2) + C$

79.  $\int \frac{u^2}{u^2 + a^2} du = u - a \tan^{-1}\left(\frac{u}{a}\right) + C$

80.  $\int \frac{u^3}{u^2 + a^2} du = \frac{u^2}{2} - \frac{a^2}{2} \ln(u^2 + a^2) + C$

81.  $\int \frac{du}{u(u^2 + a^2)} = \frac{1}{2a^2} \ln\left(\frac{u^2}{u^2 + a^2}\right) + C$

82.  $\int \frac{du}{u^2(u^2 + a^2)} = -\frac{1}{a^2 u} - \frac{1}{a^3} \tan^{-1}\left(\frac{u}{a}\right) + C$

83.  $\int \frac{du}{u^3(u^2 + a^2)} = -\frac{1}{2a^2 u^2} - \frac{1}{2a^4} \ln\left(\frac{u^2}{u^2 + a^2}\right) + C$

84.  $\int \frac{du}{(u^2 + a^2)^2} = \frac{u}{2a^2(u^2 + a^2)} + \frac{1}{2a^3} \tan^{-1}\left(\frac{u}{a}\right) + C$

85.  $\int \frac{u}{(u^2 + a^2)^2} du = -\frac{1}{2(u^2 + a^2)} + C$

86.  $\int \frac{u^2}{(u^2 + a^2)^2} du = -\frac{u}{2(u^2 + a^2)} + \frac{1}{2a} \tan^{-1}\left(\frac{u}{a}\right) + C$

87.  $\int \frac{u^3}{(u^2 + a^2)^2} du = \frac{a^2}{2(u^2 + a^2)} + \frac{1}{2} \ln(u^2 + a^2) + C$

88.  $\int \frac{du}{u(u^2 + a^2)^2} = \frac{1}{2a^2(u^2 + a^2)} + \frac{1}{2a^4} \left( \frac{u^2}{u^2 + a^2} \right) + C$

89.  $\int \frac{du}{u^2(u^2 + a^2)^2} = -\frac{1}{a^4 u} - \frac{u}{2a^4(u^2 + a^2)} - \frac{3}{2a^5} \tan^{-1}\left(\frac{u}{a}\right) + C$

90.  $\int \frac{du}{u^3(u^2 + a^2)^2} = -\frac{1}{2a^4 u^2} - \frac{1}{2a^4(u^2 + a^2)} - \frac{1}{a^6} \ln\left(\frac{u^2}{u^2 + a^2}\right) + C$

91.  $\int \frac{du}{(u^2 + a^2)^n} = -\frac{u}{2a^2(n-1)(u^2 + a^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2} \int \frac{du}{(u^2 + a^2)^{n-1}}$

92.  $\int \frac{u}{(u^2 + a^2)^n} du = -\frac{1}{2(n-1)(u^2 + a^2)^{n-1}} + C$

93.  $\int \frac{du}{u(u^2 + a^2)^n} = \frac{1}{2a^2(n-1)(u^2 + a^2)^{n-1}} + \frac{1}{a^2} \int \frac{du}{u(u^2 + a^2)^{n-1}}$

94.  $\int \frac{u^m}{(u^2 + a^2)^n} du = \int \frac{u^{m-2}}{(u^2 + a^2)^{n-1}} du - a^2 \int \frac{u^{m-2}}{(u^2 + a^2)^n} du$

95.  $\int \frac{du}{u^m(u^2 + a^2)^n} = \frac{1}{a^2} \int \frac{du}{u^m(u^2 + a^2)^{n-1}} - \frac{1}{a^2} \int \frac{du}{u^{m-2}(u^2 + a^2)^n}$

### Fórmulas con $u^2 - a^2$

96.  $\int \frac{du}{u^2 - a^2} = \begin{cases} \frac{1}{2a} \ln\left(\frac{u-a}{u+a}\right) + C \\ -\frac{1}{a} \coth^{-1}\left(\frac{u}{a}\right) + C \end{cases}$

97.  $\int \frac{u}{u^2 - a^2} du = \frac{1}{2} \ln(u^2 - a^2) + C$

98.  $\int \frac{u^2}{u^2 - a^2} du = u + \frac{a}{2} \ln\left(\frac{u-a}{u+a}\right) + C$

99.  $\int \frac{u^3}{u^2 - a^2} du = \frac{u^2}{2} + \frac{a^2}{2} \ln(u^2 - a^2) + C$

100.  $\int \frac{du}{u(u^2 - a^2)} = \frac{1}{2a^2} \ln\left(\frac{u^2 - a^2}{u^2}\right) + C$

101.  $\int \frac{du}{u^2(u^2 - a^2)} = \frac{1}{a^2 u} + \frac{1}{2a^3} \ln\left(\frac{u-a}{u+a}\right) + C$

102.  $\int \frac{du}{u^3(u^2 - a^2)} = \frac{1}{2a^2 u^2} - \frac{1}{2a^4} \ln\left(\frac{u^2}{u^2 - a^2}\right) + C$

103.  $\int \frac{du}{(u^2 - a^2)^2} = -\frac{u}{2a^2(u^2 - a^2)} - \frac{1}{4a^3} \ln\left(\frac{u-a}{u+a}\right) + C$

104.  $\int \frac{u}{(u^2 - a^2)^2} du = -\frac{1}{2(u^2 - a^2)} + C$

105.  $\int \frac{u^2}{(u^2 - a^2)^2} du = -\frac{u}{2(u^2 - a^2)} + \frac{1}{4a} \ln\left(\frac{u-a}{u+a}\right) + C$

106.  $\int \frac{u^3}{(u^2 - a^2)^2} du = -\frac{a}{2(u^2 - a^2)} + \frac{1}{2} \ln(u^2 - a^2) + C$

107.  $\int \frac{du}{u(u^2 - a^2)^2} = -\frac{1}{2a^2(u^2 - a^2)} + \frac{1}{2a^4} \ln\left(\frac{u^2}{u^2 - a^2}\right) + C$

108.  $\int \frac{du}{u^2(u^2 - a^2)^2} = -\frac{1}{a^4 u} - \frac{u}{2a^4(u^2 - a^2)} - \frac{3}{4a^5} \ln\left(\frac{u-a}{u+a}\right) + C$

109.  $\int \frac{du}{u^3(u^2 - a^2)^2} = -\frac{1}{2a^4 u^2} - \frac{1}{2a^4(u^2 - a^2)} + \frac{1}{a^6} \ln\left(\frac{u^2}{u^2 - a^2}\right) + C$

110.  $\int \frac{u}{(u^2 - a^2)^n} du = -\frac{u}{2a^2(n-1)(u^2 - a^2)^{n-1}} - \frac{2n-3}{(2n-2)a^2} \int \frac{du}{(u^2 - a^2)^{n-1}}$

$$111. \int \frac{u}{(u^2 - a^2)^n} du = -\frac{1}{2(n-1)(u^2 - a^2)^{n-1}} + C$$

$$112. \int \frac{du}{u(u^2 - a^2)^n} = -\frac{1}{2a^2(n-1)(u^2 - a^2)^{n-1}} - \frac{1}{a^2} \int \frac{du}{u(u^2 - a^2)^{n-1}}$$

$$113. \int \frac{u^m}{(u^2 - a^2)^n} du = \int \frac{u^{m-2}}{(u^2 - a^2)^{n-1}} du + a^2 \int \frac{u^{m-2}}{(u^2 - a^2)^n} du$$

$$114. \int \frac{du}{u^m(u^2 - a^2)^n} = \frac{1}{a^2} \int \frac{du}{u^{m-2}(u^2 - a^2)^n} + \frac{1}{a^2} \int \frac{du}{u^m(u^2 - a^2)^{n-1}}$$

### Fórmulas con $a^2 - u^2, u^2 < a^2$

$$115. \int \frac{du}{a^2 - u^2} = \begin{cases} \frac{1}{2a} \ln\left(\frac{a+u}{a-u}\right) + C \\ \frac{1}{a} \tanh^{-1}\left(\frac{u}{a}\right) + C \end{cases}$$

$$116. \int \frac{u}{a^2 - u^2} du = -\frac{1}{2} \ln(a^2 - u^2) + C$$

$$117. \int \frac{u^2}{a^2 - u^2} du = -u + \frac{a}{2} \ln\left(\frac{a+u}{a-u}\right) + C$$

$$118. \int \frac{u^3}{a^2 - u^2} du = -\frac{u^2}{2} - \frac{a^2}{2} \ln(a^2 - u^2) + C$$

$$119. \int \frac{du}{u(a^2 - u^2)} = \frac{1}{2a^2} \ln\left(\frac{u^2}{a^2 - u^2}\right) + C$$

$$120. \int \frac{du}{u^2(a^2 - u^2)} = \frac{1}{a^2 u} + \frac{1}{2a^3} \ln\left(\frac{a+u}{a-u}\right) + C$$

$$121. \int \frac{du}{u^3(a^2 - u^2)} = -\frac{1}{2a^2 u^2} + \frac{1}{2a^4} \ln\left(\frac{u^2}{a^2 - u^2}\right) + C$$

$$122. \int \frac{du}{(a^2 - u^2)^2} = \frac{u}{2a^2(a^2 - u^2)} + \frac{1}{4a^3} \ln\left(\frac{a+u}{a-u}\right) + C$$

$$123. \int \frac{u}{(a^2 - u^2)^3} du = \frac{1}{2(a^2 - u^2)} + C$$

$$124. \int \frac{u^2}{(a^2 - u^2)^2} du = \frac{u}{2(a^2 - u^2)} - \frac{1}{4a} \ln\left(\frac{a+u}{a-u}\right) + C$$

$$125. \int \frac{u^3}{(a^2 - u^2)^2} du = \frac{a^2}{2(a^2 - u^2)} + \frac{1}{2} \ln(a^2 - u^2) + C$$

$$126. \int \frac{du}{u(a^2 - u^2)^2} = \frac{1}{2a^2(a^2 - u^2)} + \frac{1}{2a^4} \ln\left(\frac{u^2}{a^2 - u^2}\right) + C$$

$$127. \int \frac{du}{u^2(a^2 - u^2)^2} = -\frac{1}{a^4 u} + \frac{u}{2a^4(a^2 - u^2)} + \frac{3}{4a^5} \ln\left(\frac{a+u}{a-u}\right) + C$$

$$128. \int \frac{du}{u^3(a^2 - u^2)^2} = -\frac{1}{2a^4 u^2} + \frac{1}{2a^4(a^2 - u^2)} + \frac{1}{a^6} \ln\left(\frac{u^2}{a^2 - u^2}\right) + C$$

$$129. \int \frac{du}{(a^2 - u^2)^n} = \frac{u}{2a^2(n-1)(a^2 - u^2)^{n-1}} + \frac{2n-3}{(2n-2)a^2} \int \frac{du}{(a^2 - u^2)^{n-1}}$$

$$130. \int \frac{u}{(a^2 - u^2)^n} du = \frac{1}{2a^2(n-1)(a^2 - u^2)^{n-1}} + C$$

### Fórmulas con $\sqrt{u^2 + a^2}$

$$131. \int \sqrt{u^2 + a^2} du = \frac{u\sqrt{u^2 + a^2}}{2} + \frac{a^2}{2} \ln\left(u + \sqrt{u^2 + a^2}\right) + C$$

$$132. \int u\sqrt{u^2 + a^2} du = \frac{(u^2 + a^2)^{\frac{3}{2}}}{3} + C$$

$$133. \int u^2\sqrt{u^2 + a^2} du = \frac{u(u^2 + a^2)^{\frac{3}{2}}}{4} - \frac{a^2 u \sqrt{u^2 + a^2}}{8} - \frac{a^4}{8} \ln\left(u + \sqrt{u^2 + a^2}\right) + C$$

$$134. \int u^3\sqrt{u^2 + a^2} du = \frac{(u^2 + a^2)^{\frac{5}{2}}}{5} - \frac{a^2(u^2 + a^2)^{\frac{3}{2}}}{3} + C$$

$$135. \int \frac{du}{\sqrt{u^2 + a^2}} = \begin{cases} \ln\left(u + \sqrt{u^2 + a^2}\right) + C \\ \operatorname{senh}^{-1}\left(\frac{u}{a}\right) + C \end{cases}$$

$$136. \int \frac{u}{\sqrt{u^2 + a^2}} du = \sqrt{u^2 + a^2} + C$$

$$137. \int \frac{u^2}{\sqrt{u^2 + a^2}} du = \frac{u\sqrt{u^2 + a^2}}{2} - \frac{a^2}{2} \ln\left(u + \sqrt{u^2 + a^2}\right) + C$$

$$138. \int \frac{u^3}{\sqrt{u^2 + a^2}} du = \frac{(u^2 + a^2)^{\frac{3}{2}}}{3} - a^2 \sqrt{u^2 + a^2} + C$$

$$139. \int \frac{du}{u\sqrt{u^2 + a^2}} = -\frac{1}{a} \ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right) + C$$

$$140. \int \frac{du}{u^2\sqrt{u^2 + a^2}} = -\frac{\sqrt{u^2 + a^2}}{a^2 u} + C$$

$$141. \int \frac{du}{u^3\sqrt{u^2 + a^2}} = -\frac{\sqrt{u^2 + a^2}}{2a^2 u^2} + \frac{1}{2a^3} \ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right) + C$$

$$142. \int \frac{\sqrt{u^2 + a^2}}{u} du = \sqrt{u^2 + a^2} - a \ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right) + C$$

$$143. \int \frac{\sqrt{u^2 + a^2}}{u^2} du = -\frac{\sqrt{u^2 + a^2}}{u} + \ln\left(u + \sqrt{u^2 + a^2}\right) + C$$

$$144. \int \frac{\sqrt{u^2 + a^2}}{u^3} du = -\frac{\sqrt{u^2 + a^2}}{2u^2} - \frac{1}{2a} \ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right) + C$$

$$145. \int \frac{du}{(u^2 + a^2)^{\frac{3}{2}}} = \frac{u}{a^2 \sqrt{u^2 + a^2}} + C$$

$$146. \int \frac{u}{(u^2 + a^2)^{\frac{3}{2}}} du = -\frac{1}{\sqrt{u^2 + a^2}} + C$$

$$147. \int \frac{u^2}{(u^2 + a^2)^{\frac{3}{2}}} du = -\frac{u}{\sqrt{u^2 + a^2}} + \ln(u + \sqrt{u^2 + a^2}) + C$$

$$148. \int \frac{u^3}{(u^2 + a^2)^{\frac{3}{2}}} du = \sqrt{u^2 + a^2} + \frac{a^2}{\sqrt{u^2 + a^2}} + C$$

$$149. \int \frac{du}{u(u^2 + a^2)^{\frac{3}{2}}} = \frac{1}{a^2 \sqrt{u^2 + a^2}} - \frac{1}{a^3} \ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right) + C$$

$$150. \int \frac{du}{u^2(u^2 + a^2)^{\frac{3}{2}}} = -\frac{\sqrt{u^2 + a^2}}{a^4 u} - \frac{u}{a^4 \sqrt{u^2 + a^2}} + C$$

$$151. \int \frac{du}{u^3(u^2 + a^2)^{\frac{3}{2}}} = -\frac{1}{2a^2 u^2 \sqrt{u^2 + a^2}} - \frac{3}{2a^4 \sqrt{u^2 + a^2}} + \frac{3}{2a^5}$$

$$\ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right) + C$$

$$152. \int (u^2 + a^2)^{\frac{3}{2}} du = \frac{u(u^2 + a^2)^{\frac{3}{2}}}{4} + \frac{3a^2 u \sqrt{u^2 + a^2}}{8} + \frac{3}{8} a^4 \cdot \ln(u + \sqrt{u^2 + a^2}) + C$$

$$153. \int u(u^2 + a^2)^{\frac{3}{2}} du = \frac{(u^2 + a^2)^{\frac{5}{2}}}{5} + C$$

$$154. \int u^2(u^2 + a^2)^{\frac{3}{2}} du = \frac{u(u^2 + a^2)^{\frac{5}{2}}}{6} - \frac{a^2 u(u^2 + a^2)^{\frac{3}{2}}}{24} - \frac{a^4 u \sqrt{u^2 + a^2}}{16}$$

$$-\frac{a^6}{16} \ln(u + \sqrt{u^2 + a^2}) + C$$

$$155. \int \frac{(u^2 + a^2)^{\frac{3}{2}}}{u} du = \frac{(u^2 + a^2)^{\frac{3}{2}}}{3} + a^2 \sqrt{u^2 + a^2} - a^3 \ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right) + C$$

$$156. \int \frac{(u^2 + a^2)^{\frac{3}{2}}}{u^2} du = -\frac{(u^2 + a^2)^{\frac{3}{2}}}{u} + \frac{3u \sqrt{u^2 + a^2}}{2} + \frac{3}{2} a^2$$

$$\ln(u + \sqrt{u^2 + a^2}) + C$$

$$157. \int \frac{(u^2 + a^2)^{\frac{3}{2}}}{u^3} du = -\frac{(u^2 + a^2)^{\frac{3}{2}}}{2u^2} + \frac{3}{2} \sqrt{u^2 + a^2} - \frac{3}{2} a$$

$$\ln\left(\frac{a + \sqrt{u^2 + a^2}}{u}\right) +$$

### Fórmulas con $\sqrt{u^2 - a^2}$

$$158. \int \frac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2}) + C$$

$$159. \int \frac{u}{\sqrt{u^2 - a^2}} du = \sqrt{u^2 - a^2} + C$$

$$160. \int \frac{u^2}{\sqrt{u^2 - a^2}} du = \frac{u \sqrt{u^2 - a^2}}{2} + \frac{a^2}{2} \ln(u + \sqrt{u^2 - a^2}) + C$$

$$161. \int \frac{u^3}{\sqrt{u^2 - a^2}} du = \frac{(u^2 - a^2)^{\frac{3}{2}}}{3} + a^2 \sqrt{u^2 - a^2} + C$$

$$162. \int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1}\left(\frac{u}{a}\right) + C$$

$$163. \int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

$$164. \int \frac{du}{u^3 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{2a^2 u^2} + \frac{1}{2a^3} \sec^{-1}\left(\frac{u}{a}\right) + C$$

$$165. \int \sqrt{u^2 - a^2} du = \frac{u \sqrt{u^2 - a^2}}{2} - \frac{a^2}{2} \ln(u + \sqrt{u^2 - a^2}) + C$$

$$166. \int u \sqrt{u^2 - a^2} du = \frac{(u^2 - a^2)^{\frac{3}{2}}}{3} + C$$

$$167. \int u^2 \sqrt{u^2 - a^2} du = \frac{u(u^2 - a^2)^{\frac{3}{2}}}{4} + \frac{a^2 u \sqrt{u^2 - a^2}}{8} - \frac{a^4}{8}$$

$$\ln(u + \sqrt{u^2 - a^2}) + C$$

$$168. \int u^3 \sqrt{u^2 - a^2} du = \frac{(u^2 - a^2)^{\frac{5}{2}}}{5} + \frac{a(u^2 - a^2)^{\frac{3}{2}}}{3} + C$$

$$169. \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \sec^{-1}\left(\frac{u}{a}\right) + C$$

$$170. \int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln(u + \sqrt{u^2 - a^2}) + C$$

$$171. \int \frac{\sqrt{u^2 - a^2}}{u^3} du = -\frac{\sqrt{u^2 - a^2}}{2u^2} + \frac{1}{2a} \sec^{-1}\left(\frac{u}{a}\right) + C$$

$$172. \int \frac{du}{(u^2 - a^2)^{\frac{3}{2}}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$$

$$173. \int \frac{u}{(u^2 - a^2)^{\frac{3}{2}}} du = -\frac{1}{\sqrt{u^2 - a^2}} + C$$

$$174. \int \frac{u^2}{(u^2 - a^2)^{\frac{3}{2}}} du = -\frac{u}{\sqrt{u^2 - a^2}} + \ln(u + \sqrt{u^2 - a^2}) + C$$

$$175. \int \frac{u^3}{(u^2 - a^2)^{\frac{3}{2}}} du = \sqrt{u^2 - a^2} - \frac{a^2}{\sqrt{u^2 - a^2}} + C$$

$$176. \int \frac{du}{u(u^2 - a^2)^{\frac{3}{2}}} = -\frac{1}{a^2 \sqrt{u^2 - a^2}} - \frac{1}{a^3} \sec^{-1}\left(\frac{u}{a}\right) + C$$

$$177. \int \frac{du}{u^2(u^2 - a^2)^{\frac{3}{2}}} = -\frac{\sqrt{u^2 - a^2}}{a^4 u} - \frac{u}{a^4 \sqrt{u^2 - a^2}} + C$$

$$178. \int \frac{du}{u^3(u^2 - a^2)^{\frac{3}{2}}} = \frac{1}{2a^2 u^2 \sqrt{u^2 - a^2}} - \frac{3}{2a^4 \sqrt{u^2 - a^2}} - \frac{3}{2a^5} \sec^{-1}\left(\frac{u}{a}\right) + C$$

$$179. \int (u^2 - a^2)^{\frac{3}{2}} du = 2u\sqrt{u^2 - a^2} \left( \frac{1}{8}u^2 - \frac{5}{16}a^2 \right) - \frac{3}{8}a^4 \ln(|-u + \sqrt{u^2 - a^2}|) + C$$

$$180. \int u(u^2 - a^2)^{\frac{3}{2}} du = \frac{(u^2 - a^2)^{\frac{5}{2}}}{5} + C$$

$$181. \int u^2(u^2 - a^2)^{\frac{3}{2}} du = \frac{u(u^2 - a^2)^{\frac{5}{2}}}{6} + \frac{a^2 u(u^2 - a^2)^{\frac{3}{2}}}{24} - \frac{a^4 u \sqrt{u^2 - a^2}}{16} + \frac{a^6}{16} \ln(u + \sqrt{u^2 - a^2}) + C$$

$$182. \int u^3(u^2 - a^2)^{\frac{3}{2}} du = \frac{(u^2 - a^2)^{\frac{7}{2}}}{7} + \frac{a^2(u^2 - a^2)^{\frac{5}{2}}}{5} + C$$

$$183. \int \frac{(u^2 - a^2)^{\frac{3}{2}}}{u} du = \frac{(u^2 - a^2)^{\frac{5}{2}}}{3} - a^2 \sqrt{u^2 - a^2} + a^3 \sec^{-1}\left(\frac{u}{a}\right) + C$$

$$184. \int \frac{(u^2 - a^2)^{\frac{3}{2}}}{u^2} du = -\frac{(u^2 - a^2)^{\frac{3}{2}}}{u} + \frac{3u\sqrt{u^2 - a^2}}{2} - \frac{3}{2}a^2 \ln(u + \sqrt{u^2 - a^2}) + C$$

$$185. \int \frac{(u^2 - a^2)^{\frac{3}{2}}}{u^3} du = -\frac{(u^2 - a^2)^{\frac{3}{2}}}{2u^2} + \frac{3\sqrt{u^2 - a^2}}{2} - \frac{3}{2}a \sec^{-1}\left(\frac{u}{a}\right) + C$$

**Fórmulas con  $\sqrt{a^2 - u^2}$**

$$186. \int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$187. \int \frac{u}{\sqrt{a^2 - u^2}} du = -\sqrt{a^2 - u^2} + C$$

$$188. \int \frac{u^2}{\sqrt{a^2 - u^2}} du = -\frac{u\sqrt{a^2 - u^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$189. \int \frac{u^3}{\sqrt{a^2 - u^2}} du = \frac{(a^2 - u^2)^{\frac{3}{2}}}{3} - a^2 \sqrt{a^2 - u^2} + C$$

$$190. \int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \ln\left(\frac{a + \sqrt{a^2 - u^2}}{u}\right) + C$$

$$191. \int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$$

$$192. \int \frac{du}{u^3\sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{2a^2 u^2} - \frac{1}{2a^3} \ln\left(\frac{a + \sqrt{a^2 - u^2}}{u}\right) + C$$

$$193. \int \sqrt{a^2 - u^2} du = \frac{u\sqrt{a^2 - u^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$194. \int u\sqrt{a^2 - u^2} du = -\frac{(a^2 - u^2)^{\frac{3}{2}}}{3} + C$$

$$195. \int u^2\sqrt{a^2 - u^2} du = -\frac{u(a^2 - u^2)^{\frac{3}{2}}}{4} + \frac{a^2 u \sqrt{a^2 - u^2}}{8} + \frac{a^4}{8} \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$196. \int u^3\sqrt{a^2 - u^2} du = \frac{(a^2 - u^2)^{\frac{5}{2}}}{5} - \frac{a^2(a^2 - u^2)^{\frac{3}{2}}}{3} + C$$

$$197. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln\left(\frac{a + \sqrt{a^2 - u^2}}{u}\right) + C$$

$$198. \int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{\sqrt{a^2 - u^2}}{u} - \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$199. \int \frac{\sqrt{a^2 - u^2}}{u^3} du = -\frac{\sqrt{a^2 - u^2}}{2u^2} + \frac{1}{2a} \ln\left(\frac{a + \sqrt{a^2 - u^2}}{u}\right) + C$$

$$200. \int \frac{du}{(a^2 - u^2)^{\frac{3}{2}}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

$$201. \int \frac{u}{(a^2 - u^2)^{\frac{3}{2}}} du = \frac{1}{\sqrt{a^2 - u^2}} + C$$

$$202. \int \frac{u^2}{(a^2 - u^2)^{\frac{3}{2}}} du = \frac{u}{\sqrt{a^2 - u^2}} - \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$203. \int \frac{u^3}{(a^2 - u^2)^{\frac{3}{2}}} du = \sqrt{a^2 - u^2} + \frac{a^2}{\sqrt{a^2 - u^2}} + C$$

$$204. \int \frac{du}{u(a^2 - u^2)^{\frac{3}{2}}} = \frac{1}{a^2 \sqrt{a^2 - u^2}} - \frac{1}{a^3} \ln\left(\frac{a + \sqrt{a^2 - u^2}}{u}\right) + C$$

$$205. \int \frac{du}{u^2(a^2 - u^2)^{\frac{3}{2}}} = -\frac{\sqrt{a^2 - u^2}}{a^4 u} + \frac{u}{a^4 \sqrt{a^2 - u^2}} + C$$

$$206. \int \frac{du}{u^3(a^2-u^2)^{\frac{3}{2}}} = -\frac{1}{2a^2u^2\sqrt{a^2-u^2}} + \frac{3}{2a^4\sqrt{a^2-u^2}} - \frac{3}{2a^5}$$

$$\cdot \ln\left(\frac{a+\sqrt{a^2-u^2}}{u}\right) + C$$

$$207. \int (a^2-u^2)^{\frac{3}{2}} du = \frac{u(a^2-u^2)^{\frac{3}{2}}}{4} + \frac{3a^2u\sqrt{a^2-u^2}}{8} + \frac{3}{8}a^4 \operatorname{sen}\left(\frac{u}{a}\right) + C$$

$$208. \int u(a^2-u^2)^{\frac{3}{2}} du = -\frac{(a^2-u^2)^{\frac{5}{2}}}{5} + C$$

$$209. \int u^2(a^2-u^2)^{\frac{3}{2}} du = -\frac{u(a^2-u^2)^{\frac{5}{2}}}{6} + \frac{a^2u(a^2-u^2)^{\frac{3}{2}}}{24} +$$

$$\frac{a^4u\sqrt{a^2-u^2}}{16} + \frac{a^6}{16} \operatorname{sen}^{-1}\left(\frac{u}{a}\right) + C$$

$$210. \int u^3(a^2-u^2)^{\frac{3}{2}} du = \frac{(a^2-u^2)^{\frac{7}{2}}}{7} - \frac{a^2(a^2-u^2)^{\frac{5}{2}}}{5} + C$$

$$211. \int \frac{(a^2-u^2)^{\frac{3}{2}}}{u} du = \frac{(a^2-u^2)^{\frac{3}{2}}}{3} + a^2\sqrt{a^2-u^2} - a^3$$

$$\ln\left(\frac{a+\sqrt{a^2-u^2}}{u}\right) + C$$

$$212. \int \frac{(a^2-u^2)^{\frac{3}{2}}}{u^2} du = -\frac{(a^2-u^2)^{\frac{3}{2}}}{u} - \frac{3u\sqrt{a^2-u^2}}{2} + \frac{3}{2}a^2 \operatorname{sen}\left(\frac{u}{a}\right) + C$$

$$213. \int \frac{(a^2-u^2)^{\frac{3}{2}}}{u^3} du = -\frac{(a^2-u^2)^{\frac{3}{2}}}{2u^2} - \frac{3\sqrt{a^2-u^2}}{2} + \frac{3}{2}a$$

$$\ln\left(\frac{a+\sqrt{a^2-u^2}}{u}\right) + C$$

### Fórmulas con $au^2 + bu + c$

$$214. \int \frac{du}{au^2+bu+c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \tan^{-1}\left(\frac{2au+b}{\sqrt{4ac-b^2}}\right) + C \\ \frac{1}{\sqrt{b^2-4ac}} \ln\left(\frac{2au+b-\sqrt{b^2-4ac}}{2au+b+\sqrt{b^2-4ac}}\right) + C \end{cases}$$

$$215. \int \frac{u}{au^2+bu+c} du = \frac{1}{2a} \ln(au^2+bu+c) - \frac{b}{2a} \int \frac{du}{au^2+bu+c}$$

$$216. \int \frac{u^2}{au^2+bu+c} du = \frac{u}{a} - \frac{b}{2a^2} \ln(au^2+bu+c) + \frac{b^2-2ac}{2a^2} \int \frac{du}{au^2+bu+c}$$

$$217. \int \frac{du}{u(au^2+bu+c)} = \frac{1}{2c} \ln\left(\frac{u^2}{au^2+bu+c}\right) - \frac{b}{2c} \int \frac{du}{au^2+bu+c}$$

$$218. \int \frac{du}{u^2(au^2+bu+c)} = \frac{b}{2c^2} \ln\left(\frac{au^2+bu+c}{u^2}\right) - \frac{1}{cu} + \frac{b^2-2ac}{2c^2} \int \frac{du}{au^2+bu+c}$$

$$219. \int \frac{du}{(au^2+bu+c)^2} = \frac{2au+b}{(4ac-b^2)(au^2+bu+c)} + \frac{2a}{4ac-b^2} \int \frac{du}{au^2+bu+c}$$

$$220. \int \frac{u}{(au^2+bu+c)^2} du = -\frac{bu+2c}{(4ac-b^2)(au^2+bu+c)} - \frac{b}{4ac-b^2} \int \frac{du}{au^2+bu+c}$$

$$221. \int \frac{u^2}{(au^2+bu+c)^2} du = \frac{(b^2-2ac)u+bc}{a(4ac-b^2)(au^2+bu+c)} + \frac{2c}{4ac-b^2} \int \frac{du}{au^2+bu+c}$$

$$222. \int \frac{du}{u(au^2+bu+c)^2} = \frac{1}{2c(au^2+bu+c)} - \frac{b}{2c} \int \frac{du}{(au^2+bu+c)^2} + \frac{1}{c} \int \frac{du}{u(au^2+bu+c)}$$

$$223. \int \frac{du}{u^2(au^2+bu+c)^2} = -\frac{1}{cu(au^2+bu+c)} - \frac{3a}{c} \int \frac{du}{(au^2+bu+c)^2} - \frac{2b}{c} \int \frac{du}{u(au^2+bu+c)^2}$$

$$224. \int \frac{u^m}{au^2+bu+c} du = \frac{u^{m-1}}{a(m-1)} - \frac{c}{a} \int \frac{u^{m-2}}{au^2+bu+c} du - \frac{b}{a} \int \frac{u^{m-1}}{au^2+bu+c} du$$

$$225. \int \frac{du}{u^n(au^2+bu+c)} = -\frac{1}{c(n-1)u^{n-1}} - \frac{b}{c} \int \frac{du}{u^{n-1}(au^2+bu+c)} - \frac{a}{c} \int \frac{du}{u^{n-2}(au^2+bu+c)}$$

### Fórmulas con $u^3 + a^3$

$$226. \int \frac{du}{u^3+a^3} = \frac{1}{6a^2} \ln\left(\frac{(a+u)^2}{u^2-au+a^2}\right) + \frac{1}{a^2\sqrt{3}} \tan^{-1}\left(\frac{2u-a}{a\sqrt{3}}\right) + C$$

$$227. \int \frac{u}{u^3+a^3} du = \frac{1}{6a} \ln\left(\frac{u^2-au+a^2}{(u+a)^2}\right) + \frac{1}{a\sqrt{3}} \tan^{-1}\left(\frac{2u-a}{a\sqrt{3}}\right) + C$$

$$228. \int \frac{u^2}{u^3+a^3} du = \frac{1}{3} \ln(u^3+a^3) + C$$

$$229. \int \frac{du}{u(u^3+a^3)} = \frac{1}{3a^3} \ln\left(\frac{u^3}{u^3+a^3}\right) + C$$

$$230. \int \frac{du}{u^2(u^3+a^3)} = -\frac{1}{a^3u} - \frac{1}{6a^4} \ln\left(\frac{u^2-au+a^2}{(u+a)^2}\right) - \frac{1}{a^4\sqrt{3}} \tan^{-1}\left(\frac{2u-a}{a\sqrt{3}}\right) + C$$

$$231. \int \frac{du}{(u^3+a^3)^2} = \frac{u}{3a^3(u^3+a^3)} + \frac{1}{9a^5} \ln\left(\frac{(u+a)^2}{u^2-au+a^2}\right) + \frac{2}{3a^5\sqrt{3}} \tan^{-1}\left(\frac{2u-a}{a\sqrt{3}}\right) + C$$

$$232. \int \frac{u}{(u^3+a^3)^2} du = \frac{u^2}{3a^5(u^3+a^3)} + \frac{1}{18a^4} \ln\left(\frac{u^2-au+a^2}{(u+a)^2}\right) + \frac{1}{3a^4\sqrt{3}} \tan^{-1}\left(\frac{2u-a}{a\sqrt{3}}\right) + C$$

$$233. \int \frac{u^2}{(u^3+a^3)^2} du = -\frac{1}{3(u^3+a^3)} + C$$

$$234. \int \frac{du}{u(u^3+a^3)^2} = \frac{1}{3a^3(u^3+a^3)} + \frac{1}{3a^6} \ln\left(\frac{u^3}{u^3+a^3}\right) + C$$

$$235. \int \frac{du}{u^2(u^3+a^3)^2} = -\frac{1}{a^6u} - \frac{u^2}{3a^6(u^3+a^3)} - \frac{4}{3a^6} \int \frac{u}{u^3+a^3} du$$

$$236. \int \frac{u^m}{u^3+a^3} du = \frac{u^{m-2}}{m-2} - a^3 \int \frac{u^{m-3}}{u^3+a^3} du$$

$$237. \int \frac{du}{u^n(u^3+a^3)} = -\frac{1}{a^3(n-1)} u^{n-1} - \frac{1}{a^3} \int \frac{du}{u^{n-3}(u^3+a^3)}$$

### Fórmulas con $u^4 \pm a^4$

$$238. \int \frac{du}{u^4+a^4} = \frac{1}{4a^3\sqrt{2}} \ln\left(\frac{u^2+au\sqrt{2}+a^2}{u^2-au\sqrt{2}+a^2}\right) - \frac{1}{2a^3\sqrt{2}} \left[ \tan^{-1}\left(1-\frac{u\sqrt{2}}{a}\right) - \tan^{-1}\left(1+\frac{u\sqrt{2}}{a}\right) \right] + C$$

$$239. \int \frac{u^2}{u^4+a^4} du = \frac{1}{4a\sqrt{2}} \ln\left(\frac{u^2-au\sqrt{2}+a^2}{u^2+au\sqrt{2}+a^2}\right) - \frac{1}{2a\sqrt{2}} \left[ \tan^{-1}\left(1-\frac{u\sqrt{2}}{a}\right) - \tan^{-1}\left(1+\frac{u\sqrt{2}}{a}\right) \right] + C$$

$$240. \int \frac{du}{u^2(u^4+a^4)} = -\frac{1}{a^4u} - \frac{1}{4a^5\sqrt{2}} \ln\left(\frac{u^2-au\sqrt{2}+a^2}{u^2+au\sqrt{2}+a^2}\right) + \frac{1}{2a^5\sqrt{2}} \left[ \tan^{-1}\left(1-\frac{u\sqrt{2}}{a}\right) - \tan^{-1}\left(1+\frac{u\sqrt{2}}{a}\right) \right] + C$$

$$241. \int \frac{u^3}{u^4+a^4} du = \frac{1}{4} \ln(u^4+a^4) + C$$

$$242. \int \frac{du}{u(u^4+a^4)} = \frac{1}{4a^2} \ln\left(\frac{u^4}{u^4+a^4}\right) + C$$

$$243. \int \frac{u}{u^4+a^4} du = \frac{1}{2a^2} \tan^{-1}\left(\frac{u^2}{a^2}\right) + C$$

$$244. \int \frac{du}{u^3(u^4+a^4)} = -\frac{1}{2a^4u^2} - \frac{1}{2a^6} \tan^{-1}\left(\frac{u^2}{a^2}\right) + C$$

$$245. \int \frac{du}{u^4-a^4} = \frac{1}{4a^3} \ln\left(\frac{u-a}{u+a}\right) - \frac{1}{2a^3} \tan^{-1}\left(\frac{u}{a}\right) + C$$

$$246. \int \frac{u}{u^4-a^4} du = \frac{1}{4a^2} \ln\left(\frac{u^2-a^2}{u^2+a^2}\right) + C$$

$$247. \int \frac{u^2}{u^4-a^4} du = \frac{1}{4a} \ln\left(\frac{u-a}{u+a}\right) + \frac{1}{2a} \tan^{-1}\left(\frac{u}{a}\right) + C$$

$$248. \int \frac{u^3}{u^4-a^4} du = \frac{1}{4} \ln(u^4-a^4) + C$$

$$249. \int \frac{du}{u(u^4-a^4)} = \frac{1}{4a^4} \ln\left(\frac{u^4-a^4}{u^4}\right) + C$$

$$250. \int \frac{du}{u^2(u^4-a^4)} = \frac{1}{a^4u} + \frac{1}{4a^5} \ln\left(\frac{u-a}{u+a}\right) + \frac{1}{2a^5} \tan^{-1}\left(\frac{u}{a}\right) + C$$

$$251. \int \frac{du}{u^3(u^4-a^4)} = \frac{1}{2a^4u^2} + \frac{1}{4a^6} \ln\left(\frac{u^2-a^2}{u^2+a^2}\right) + C$$

### Fórmulas con $\sin(au)$

$$252. \int \sin(au) du = -\frac{\cos(au)}{a} + C$$

$$253. \int u \sin(au) du = \frac{\sin(au)}{a^2} - \frac{u \cos(au)}{a} + C$$

$$254. \int u^2 \sin(au) du = \frac{2u}{a^2} \sin(au) + \left( \frac{2}{a^3} - \frac{u^2}{a} \right) \cos(au) + C$$

$$255. \int u^3 \sin(au) du = \left( \frac{3u^2}{a^2} - \frac{6}{a^4} \right) \sin(au) + \left( \frac{6u}{a^3} - \frac{u^3}{a} \right) \cos(au) + C$$

$$256. \int u^n \sin(au) du = -\frac{u^n \cos(au)}{a} + \frac{n}{a} \int u^{n-1} \cos(au) du$$

$$257. \int u^n \sin(au) du = -\frac{u^n \cos(au)}{a} + \frac{n u^{n-1}}{a^2} \sin(au) - \frac{n(n-1)}{a^2}$$

$$\int u^{n-2} \sin(au) du$$

$$258. \int \sin^2(au) du = \frac{u}{2} - \frac{\sin(2au)}{4a} + C$$

$$259. \int \operatorname{sen}^3(au)du = -\frac{\cos(au)}{a} + \frac{\cos^3(au)}{3a} + C$$

$$260. \int \operatorname{sen}^4(au)du = \frac{3u}{8} - \frac{\operatorname{sen}(2au)}{4a} + \frac{\operatorname{sen}(4au)}{32a} + C$$

$$261. \int u \operatorname{sen}^2(au)du = \frac{u^2}{4} - \frac{u \operatorname{sen}(2au)}{4a} - \frac{\cos(au)}{8a^2} + C$$

$$262. \int \frac{\operatorname{sen}(au)}{u}du = au - \frac{(au)^3}{3 \cdot 3!} + \frac{(au)^5}{5 \cdot 5!} - \dots$$

$$263. \int \frac{\operatorname{sen}(au)}{u^2}du = -\frac{\operatorname{sen}(au)}{u} + a \int \frac{\cos(au)}{u}du$$

$$264. \int \frac{du}{\operatorname{sen}(au)} = \begin{cases} \frac{1}{a} \ln(\csc(au) - \cot(au)) + C \\ \frac{1}{a} \ln\left(\tan\left(\frac{au}{2}\right)\right) + C \end{cases}$$

$$265. \int \frac{u}{\operatorname{sen}(au)}du = \frac{1}{a^2} \left\{ au + \frac{(au)^3}{18} + \frac{7(au)^5}{1800} + \dots + \frac{2(2^{2n-1}-1)B_n(au)^{2n+1}}{(2n+1)!} + \dots \right\}$$

$$266. \int \frac{du}{\operatorname{sen}^2(au)} = -\frac{1}{a} \cot(au) + C$$

$$267. \int \frac{du}{\operatorname{sen}^3(au)} = -\frac{\cos(au)}{2a \operatorname{sen}^2(au)} + \frac{1}{2a} \ln\left[\tan\left(\frac{au}{2}\right)\right] + C$$

$$268. \int \operatorname{sen}(au) \operatorname{sen}(bu)du = \frac{\operatorname{sen}((a-b)u)}{2(a-b)} - \frac{\operatorname{sen}((a+b)u)}{2(a+b)} + C \quad a \neq b$$

$$269. \int \frac{du}{1-\operatorname{sen}(au)} = \frac{1}{a} \tan\left(\frac{\pi}{4} + \frac{au}{2}\right) + C$$

$$270. \int \frac{u}{1-\operatorname{sen}(au)}du = \frac{u}{a} \tan\left(\frac{\pi}{4} + \frac{au}{2}\right) + \frac{2}{a^2} \ln\left[\operatorname{sen}\left(\frac{\pi}{4} + \frac{au}{2}\right)\right] + C$$

$$271. \int \frac{du}{1+\operatorname{sen}(au)} = -\frac{1}{a} \tan\left(\frac{\pi}{4} - \frac{au}{2}\right) + C$$

$$272. \int \frac{u}{1+\operatorname{sen}(au)}du = -\frac{u}{a} \tan\left(\frac{\pi}{4} - \frac{au}{2}\right) + \frac{2}{a^2} \ln\left[\operatorname{sen}\left(\frac{\pi}{4} + \frac{au}{2}\right)\right] + C$$

$$273. \int \frac{du}{(1-\operatorname{sen}(au))^2} = \frac{1}{2a} \tan\left(\frac{\pi}{4} + \frac{au}{2}\right) + \frac{1}{6a} \tan^3\left(\frac{\pi}{4} + \frac{au}{2}\right) + C$$

$$274. \int \frac{du}{(1+\operatorname{sen}(au))^2} = -\frac{1}{2a} \tan\left(\frac{\pi}{4} - \frac{au}{2}\right) - \frac{1}{6a} \tan^3\left(\frac{\pi}{4} - \frac{au}{2}\right) + C$$

## Fórmulas con cos (au)

$$275. \int \cos(au)du = \frac{\operatorname{sen}(au)}{a} + C$$

$$276. \int u \cos(au)du = \frac{\cos(au)}{a^2} + \frac{u \operatorname{sen}(au)}{a} + C$$

$$277. \int u^2 \cos(au)du = \frac{2u}{a^2} \cos(au) + \left(\frac{u^2}{a} - \frac{2}{a^3}\right) \operatorname{sen}(au) + C$$

$$278. \int u^3 \cos(au)du = \left(\frac{3u^2}{a^2} - \frac{6}{a^4}\right) \cos(au) + \left(\frac{u^3}{a} - \frac{6u}{a^3}\right) \operatorname{sen}(au) + C$$

$$279. \int u^n \cos(au)du = \frac{u^n \operatorname{sen}(au)}{a} - \frac{n}{a} \int u^{n-1} \operatorname{sen}(au)du$$

$$280. \int u^n \cos(au)du = -\frac{u^n \operatorname{sen}(au)}{a} + \frac{n u^{n-1}}{a^2} \cos(au) - \frac{n(n-1)}{a^2} \int u^{n-2} \cos(au)du$$

$$281. \int \cos^2(au)du = \frac{u}{2} + \frac{\operatorname{sen}(2au)}{4a} + C$$

$$282. \int \cos^3(au)du = \frac{\operatorname{sen}(au)}{a} - \frac{\operatorname{sen}^3(au)}{3a} + C$$

$$283. \int \cos^4(au)du = \frac{3u}{8} + \frac{\operatorname{sen}(2au)}{4a} + \frac{\operatorname{sen}(4au)}{32a} + C$$

$$284. \int u \cos^2(au)du = \frac{u^2}{4} + \frac{u \operatorname{sen}(2au)}{4a} + \frac{\cos(2au)}{8a^2} + C$$

$$285. \int \frac{\cos(au)}{u}du = \ln(u) - \frac{(au)^2}{2 \cdot 2!} + \frac{(au)^4}{4 \cdot 4!} - \frac{(au)^6}{6 \cdot 6!} + \dots$$

$$286. \int \frac{\cos(au)}{u^2}du = -\frac{\cos(au)}{u} - a \int \frac{\operatorname{sen}(au)}{u}du$$

$$287. \int \frac{du}{\cos(au)} = \begin{cases} \frac{1}{a} \ln[\sec(au) + \tan(au)] + C \\ \frac{1}{a} \ln\left[\tan\left(\frac{\pi}{4} + \frac{au}{2}\right)\right] + C \end{cases}$$

$$288. \int \frac{u}{\cos(au)}du = \frac{1}{a^2} \left\{ \frac{(au)^2}{2} + \frac{(au)^4}{8} + \frac{5(au)^6}{144} + \dots + \frac{E_n(au)^{2n+2}}{(2n+2)(2n)!} + \dots \right\}$$

$$289. \int \frac{du}{\cos^2(au)} = \frac{\tan(au)}{a} + C$$

$$290. \int \frac{du}{\cos^3(au)} = \frac{\operatorname{sen}(au)}{2a \cos^2(au)} + \frac{1}{2a} \ln\left[\tan\left(\frac{\pi}{4} + \frac{au}{2}\right)\right] + C$$

$$291. \int \cos(au)\cos(bu)du = \frac{\operatorname{sen}((a-b)u)}{2(a-b)} - \frac{\operatorname{sen}((a+b)u)}{2(a+b)} + C \quad a \neq b$$

$$292. \int \frac{du}{1-\cos(au)} = -\frac{1}{a}\cot\left(\frac{au}{2}\right) + C$$

$$293. \int \frac{u}{1-\cos(au)}du = -\frac{u}{a}\cot\left(\frac{au}{2}\right) + \frac{2}{a^2}\ln\left[\operatorname{sen}\left(\frac{au}{2}\right)\right] + C$$

$$294. \int \frac{du}{1+\cos(au)} = \frac{1}{a}\tan\left(\frac{au}{2}\right) + C$$

$$295. \int \frac{u}{1+\cos(au)}du = \frac{u}{a}\tan\left(\frac{au}{2}\right) + \frac{2}{a^2}\ln\left[\cos\left(\frac{au}{2}\right)\right] + C$$

$$296. \int \frac{du}{(1-\cos(au))^2} = -\frac{1}{2a}\cot\left(\frac{au}{2}\right) - \frac{1}{6a}\cot^3\left(\frac{au}{2}\right) + C$$

$$297. \int \frac{du}{(1+\cos(au))^2} = \frac{1}{2a}\tan\left(\frac{au}{2}\right) + \frac{1}{6a}\tan^3\left(\frac{au}{2}\right) + C$$

### Fórmulas con $\cos(au)$ y $\operatorname{sen}(au)$

$$298. \int \operatorname{sen}(au)\cos(au)du = \frac{\operatorname{sen}^2(au)}{2a} + C$$

$$299. \int \operatorname{sen}(pu)\cos(qu)du = -\frac{\cos[(p-q)u]}{2(p-q)} - \frac{\cos[(p+q)u]}{2(p+q)} + C$$

$$300. \int \operatorname{sen}^n(au)\cos(au)du = \frac{\operatorname{sen}^{n+1}(au)}{a(n+1)} + C$$

$$301. \int \cos^n(au)\operatorname{sen}(au)du = -\frac{\cos^{n+1}(au)}{a(n+1)} + C$$

$$302. \int \operatorname{sen}^2(au)\cos^2(au)du = \frac{u}{8} - \frac{\operatorname{sen}(4au)}{32a} + C$$

$$303. \int \frac{du}{\operatorname{sen}(au)\cos(au)} = \frac{1}{a}\ln\left[\tan(au)\right] + C$$

$$304. \int \frac{du}{\operatorname{sen}^2(au)\cos(au)} = \frac{1}{a}\ln\left[\tan\left(\frac{\pi}{4} + \frac{au}{2}\right)\right] - \frac{1}{a\operatorname{sen}(au)} + C$$

$$305. \int \frac{du}{\operatorname{sen}(au)\cos^2(au)} = \frac{1}{a}\ln\left[\tan\left(\frac{au}{2}\right)\right] + \frac{1}{a\cos(au)} + C$$

$$306. \int \frac{du}{\operatorname{sen}^2(au)\cos^2(au)} = -\frac{2\cot(2au)}{a} + C$$

$$307. \int \frac{\operatorname{sen}^2(au)}{\cos(au)}du = -\frac{\operatorname{sen}(au)}{a} + \frac{1}{a}\ln\left[\tan\left(\frac{\pi}{4} + \frac{au}{2}\right)\right] + C$$

$$308. \int \frac{\cos^2(au)}{\operatorname{sen}(au)}du = \frac{\cos(au)}{a} + \frac{1}{a}\ln\left[\tan\left(\frac{au}{2}\right)\right] + C$$

$$309. \int \frac{du}{\operatorname{sen}(au) \pm \cos(au)} = \frac{1}{a\sqrt{2}}\ln\left[\tan\left(\frac{\pi}{8} \pm \frac{au}{2}\right)\right] + C$$

$$310. \int \frac{\operatorname{sen}(au)}{\operatorname{sen}(au) \pm \cos(au)}du = \frac{u}{2} \mp \frac{1}{2a}\ln[\operatorname{sen}(au) \pm \cos(au)] + C$$

$$311. \int \frac{\cos(au)}{\operatorname{sen}(au) \pm \cos(au)}du = \frac{1}{2a}\ln[\operatorname{sen}(au) \pm \cos(au)] \pm \frac{u}{2} + C$$

### Fórmulas con $\tan(au)$

$$312. \int \tan(au)du = \begin{cases} -\frac{1}{a}\ln[\cos(au)] + C \\ \frac{1}{a}\ln[\sec(au)] + C \end{cases}$$

$$313. \int \tan^2(au)du = \frac{\tan(au)}{a} - u + C$$

$$314. \int \tan^3(au)du = \frac{\tan^2(au)}{2a} + \frac{1}{a}\ln[\cos(au)] + C$$

$$315. \int \tan^n(au)du = \frac{\tan^{n-1}(au)}{a(n-1)} - \int \tan^{n-2}(au)du$$

$$316. \int \tan^n(au)\sec^2(au)du = \frac{\tan^{n+1}(au)}{a(n+1)} + C$$

$$317. \int \frac{\sec^2(au)}{\tan(au)}du = \frac{1}{a}\ln[\tan(au)] + C$$

$$318. \int \frac{du}{\tan(au)} = \frac{1}{a}\ln[\operatorname{sen}(au)] + C$$

$$319. \int u\tan^2(au)du = \frac{u\tan(au)}{a} + \frac{1}{a^2}\ln[\cos(au)] - \frac{u^2}{2} + C$$

### Fórmulas con $\cot(au)$

$$320. \int \cot(au)du = \frac{1}{a}\ln[\operatorname{sen}(au)] + C$$

$$321. \int \cot^2(au)du = -\frac{\cot(au)}{a} - u + C$$

$$322. \int \cot^3(au)du = -\frac{\cot^2(au)}{2a} - \frac{1}{a}\ln[\operatorname{sen}(au)] + C$$

$$323. \int \cot^n(au)\csc^2(au)du = -\frac{\cot^{n-1}(au)}{a(n+1)} + C$$

$$324. \int \frac{\csc^2(au)}{\cot(au)}du = -\frac{1}{a}\ln[\cot(au)] + C$$

$$325. \int \frac{du}{\cot(au)} = -\frac{1}{a} \ln[\cos(au)] + C$$

$$326. \int u \cot^2(au) du = -\frac{u \cot(au)}{a} + \frac{1}{a^2} \ln[\sin(au)] - \frac{u^2}{2} + C$$

$$327. \int \cot^n(au) du = -\frac{\cot^{n-1}(au)}{a(n-1)} - \int \cot^{n-2}(au) du$$

### Fórmulas con $\sec(au)$

$$328. \int \sec(au) du = \begin{cases} \frac{1}{a} \ln[\sec(au) + \tan(au)] + C \\ \frac{1}{a} \ln\left[\tan\left(\frac{\pi}{4} + \frac{au}{2}\right)\right] + C \end{cases}$$

$$329. \int \sec^2(au) du = \frac{\tan(au)}{a} + C$$

$$330. \int \sec^3(au) du = \frac{\sec(au) \tan(au)}{2a} + \frac{1}{2a} \ln[\sec(au) + \tan(au)] + C$$

$$331. \int \sec^n(au) \tan(au) du = \frac{\sec^n(au)}{an} + C$$

$$332. \int \frac{du}{\sec(au)} = \frac{\sin(au)}{a} + C$$

$$333. \int u \sec^2(au) du = \frac{u}{a} \tan(au) + \frac{1}{a^2} \ln[\cos(au)] + C$$

$$334. \int \sec^n(au) du = \frac{\sec^{n-2}(au) \tan(au)}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2}(au) du$$

### Fórmulas con $\csc(au)$

$$335. \int \csc(au) du = \begin{cases} \frac{1}{a} \ln[\csc(au) - \cot(au)] + C \\ \frac{1}{a} \ln\left[\tan\left(\frac{au}{2}\right)\right] + C \end{cases}$$

$$336. \int \csc^2(au) du = -\frac{\cot(au)}{a} + C$$

$$337. \int \csc^3(au) du = -\frac{\csc(au) \cot(au)}{2a} + \frac{1}{2a} \ln\left[\tan\left(\frac{au}{2}\right)\right] + C$$

$$338. \int \csc^n(au) \cot(au) du = -\frac{\csc^n(au)}{na} + C$$

$$339. \int \frac{du}{\csc(au)} = -\frac{\cos(au)}{a} + C$$

$$340. \int u \csc^2(au) du = -\frac{u \cot(au)}{a} + \frac{1}{a^2} \ln[\sin(au)] + C$$

$$341. \int \csc^n(au) du = -\frac{\csc^{n-2}(au) \cot(au)}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2}(au) du$$

### Fórmulas con funciones trigonométricas inversas

$$342. \int \sin^{-1}\left(\frac{u}{a}\right) du = u \sin^{-1}\left(\frac{u}{a}\right) + \sqrt{a^2 - u^2} + C$$

$$343. \int u \sin^{-1}\left(\frac{u}{a}\right) du = \left(\frac{u^2}{2} - \frac{a^2}{4}\right) \sin^{-1}\left(\frac{u}{a}\right) + \frac{u \sqrt{a^2 - u^2}}{4} + C$$

$$344. \int u^2 \sin^{-1}\left(\frac{u}{a}\right) du = \frac{u^3}{3} \sin^{-1}\left(\frac{u}{a}\right) + \frac{(u^2 + 2a^2)\sqrt{a^2 - u^2}}{9} + C$$

$$345. \int \frac{\sin^{-1}\left(\frac{u}{a}\right)}{u} du = \frac{u}{a} + \frac{\left(\frac{u}{a}\right)^3}{2 \cdot 3 \cdot 3} + \frac{1 \cdot 3 \cdot 5 \left(\frac{u}{a}\right)^5}{2 \cdot 4 \cdot 5 \cdot 5} + \frac{1 \cdot 3 \cdot 5 \cdot 7 \left(\frac{u}{a}\right)^7}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} + \dots$$

$$346. \int \frac{\sin^{-1}\left(\frac{u}{a}\right)}{u^2} du = -\frac{\sin^{-1}\left(\frac{u}{a}\right)}{u} - \frac{1}{a} \ln\left[\frac{a + \sqrt{a^2 - u^2}}{u}\right] + C$$

$$347. \int \left[\sin^{-1}\left(\frac{u}{a}\right)\right]^2 dx = u \left(\sin^{-1}\left(\frac{u}{a}\right)\right)^2 - 2u + 2\sqrt{a^2 - u^2} \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$348. \int \cos^{-1}\left(\frac{u}{a}\right) du = u \cos^{-1}\left(\frac{u}{a}\right) - \sqrt{a^2 - u^2} + C$$

$$349. \int u \cos^{-1}\left(\frac{u}{a}\right) du = \left(\frac{u^2}{2} - \frac{a^2}{4}\right) \cos^{-1}\left(\frac{u}{a}\right) - \frac{u \sqrt{a^2 - u^2}}{4} + C$$

$$350. \int u^2 \cos^{-1}\left(\frac{u}{a}\right) du = \frac{u^3}{3} \cos^{-1}\left(\frac{u}{a}\right) - \frac{(u^2 - 2a^2)\sqrt{a^2 - u^2}}{9} + C$$

$$351. \int \frac{\cos^{-1}\left(\frac{u}{a}\right)}{u} du = \frac{\pi}{2} \ln(u) - \int \frac{\sin^{-1}\left(\frac{u}{a}\right)}{u} du$$

$$352. \int \frac{\cos^{-1}\left(\frac{u}{a}\right)}{u^2} du = -\frac{\cos^{-1}\left(\frac{u}{a}\right)}{u} + \frac{1}{a} \ln\left(\frac{a + \sqrt{a^2 - u^2}}{u}\right) + C$$

$$353. \int \left[\cos^{-1}\left(\frac{u}{a}\right)\right]^2 du = u \left(\cos^{-1}\left(\frac{u}{a}\right)\right)^2 - 2u - 2\sqrt{a^2 - u^2} \cos^{-1}\left(\frac{u}{a}\right) + C$$

$$354. \int \tan^{-1}\left(\frac{u}{a}\right) du = u \tan^{-1}\left(\frac{u}{a}\right) - \frac{a}{2} \ln(u^2 + a^2) + C$$

**355.**  $\int u \tan^{-1}\left(\frac{u}{a}\right) du = \frac{1}{2}(u^2 + a^2) \tan^{-1}\left(\frac{u}{a}\right) - \frac{au}{2} + C$

**356.**  $\int u^2 \tan^{-1}\left(\frac{u}{a}\right) du = \frac{u^3}{3} \tan^{-1}\left(\frac{u}{a}\right) - \frac{au^2}{6} + \frac{a^3}{6} \ln(u^2 + a^2) + C$

**357.**  $\int \frac{\tan^{-1}\left(\frac{u}{a}\right)}{u} du = \frac{u}{a} - \frac{\left(\frac{u}{a}\right)^3}{3^2} + \frac{\left(\frac{u}{a}\right)^5}{5^2} - \frac{\left(\frac{u}{a}\right)^7}{7^2} + \dots$

**358.**  $\int \frac{\tan^{-1}\left(\frac{u}{a}\right)}{u^2} du = -\frac{u}{a} \tan^{-1}\left(\frac{u}{a}\right) - \frac{1}{2a} \ln\left(\frac{u^2 + a^2}{u^2}\right) + C$

**359.**  $\int \cot^{-1}\left(\frac{u}{a}\right) du = u \cot^{-1}\left(\frac{u}{a}\right) + \frac{a}{2} \ln(u^2 + a^2) + C$

**360.**  $\int u \cot^{-1}\left(\frac{u}{a}\right) du = \frac{1}{2}(u^2 + a^2) \cot^{-1}\left(\frac{u}{a}\right) + \frac{au}{2} + C$

**361.**  $\int u^2 \cot^{-1}\left(\frac{u}{a}\right) du = \frac{u^3}{3} \cot^{-1}\left(\frac{u}{a}\right) + \frac{au^2}{6} - \frac{a^3}{6} \ln(u^2 + a^2) + C$

**362.**  $\int \frac{\cot^{-1}\left(\frac{u}{a}\right)}{u} du = \frac{\pi}{2} \ln(u) - \int \frac{\tan^{-1}\left(\frac{u}{a}\right)}{u} du$

**363.**  $\int \frac{\cot^{-1}\left(\frac{u}{a}\right)}{u^2} du = -\frac{\cot^{-1}\left(\frac{u}{a}\right)}{u} + \frac{1}{2a} \ln\left(\frac{u^2 + a^2}{u^2}\right) + C$

**364.**  $\int u^m \operatorname{sen}^{-1}\left(\frac{u}{a}\right) du = \frac{u^{m+1}}{m+1} \operatorname{sen}^{-1}\left(\frac{u}{a}\right) - \frac{1}{m+1} \int \frac{u^{m+1}}{\sqrt{a^2 - u^2}} du$

**365.**  $\int u^m \cos^{-1}\left(\frac{u}{a}\right) du = \frac{u^{m+1}}{m+1} \cos^{-1}\left(\frac{u}{a}\right) + \frac{1}{m+1} \int \frac{u^{m+1}}{\sqrt{a^2 - u^2}} du$

**366.**  $\int u^m \tan^{-1}\left(\frac{u}{a}\right) du = \frac{u^{m+1}}{m+1} \tan^{-1}\left(\frac{u}{a}\right) - \frac{a}{m+1} \int \frac{u^{m+1}}{u^2 + a^2} du$

**367.**  $\int u^m \cot^{-1}\left(\frac{u}{a}\right) du = \frac{u^{m+1}}{m+1} \cot^{-1}\left(\frac{u}{a}\right) + \frac{a}{m+1} \int \frac{u^{m+1}}{u^2 + a^2} du$

### Fórmulas con $e^{au}$

**368.**  $\int e^{au} du = \frac{e^{au}}{a} + C$

**369.**  $\int ue^{au} du = \frac{e^{au}}{a} \left( u - \frac{1}{a} \right) + C$

**370.**  $\int u^2 e^{au} du = \frac{e^{au}}{a} \left( u^2 - \frac{2u}{a} + \frac{2}{a^2} \right) + C$

**371.**

$$\int u^n e^{au} du = \begin{cases} \frac{u^n e^{au}}{a} - \frac{n}{a} \int u^{n-1} e^{au} du \\ \frac{e^{au}}{a} \left( u^n - \frac{nu^{n-1}}{a} + \frac{n(n-1)u^{n-2}}{a^2} + \dots + \frac{(-1)^n n!}{a^n} \right) + C \quad \forall n \in \mathbb{N} \end{cases}$$

**372.**  $\int \frac{e^{au}}{u} du = \ln(u) + \frac{au}{1 \cdot 1!} + \frac{(au)^2}{2 \cdot 2!} + \frac{(au)^3}{3 \cdot 3!} + \dots$

**373.**  $\int \frac{e^{au}}{u^n} du = -\frac{e^{au}}{(n-1)u^{n-1}} + \frac{a}{n-1} \int \frac{e^{au}}{u^{n-1}} du$

**374.**  $\int \frac{du}{p + pe^{au}} = \frac{u}{p} - \frac{1}{ap} \ln(p + pe^{au}) + C$

**375.**  $\int \frac{du}{(p + pe^{au})^2} = \frac{u}{p^2} + \frac{1}{ap(p + pe^{au})} - \frac{1}{ap^2} \ln(p + pe^{au}) + C$

**376.**  $\int \frac{du}{pe^{au} + qe^{-au}} = \begin{cases} \frac{1}{a\sqrt{pq}} \tan^{-1}\left(\sqrt{\frac{p}{q}} e^{au}\right) + C \\ \frac{1}{2a\sqrt{-pq}} \ln\left(\frac{e^{au} - \sqrt{-\frac{q}{p}}}{e^{au} + \sqrt{-\frac{q}{p}}}\right) + C \end{cases}$

**377.**  $\int e^{au} \operatorname{sen}(bu) du = \frac{e^{au} [\operatorname{asen}(bu) - b \cos(bu)]}{a^2 - b^2} + C$

**378.**  $\int e^{au} \cos(bu) du = \frac{e^{au} [a \cos(bu) + b \operatorname{sen}(bu)]}{a^2 + b^2} + C$

**379.**  $\int e^{au} \ln(u) du = \frac{e^{au} \ln(u)}{a} - \frac{1}{a} \int \frac{e^{au}}{u} du$

### Fórmulas con $\ln(u)$

**380.**  $\int \ln(u) du = u \ln(u) - u + C$

**381.**  $\int [\ln(u)]^2 du = u [\ln(u)]^2 - 2u \ln(u) + 2u + C$

**382.**  $\int [\ln(u)]^n du = u [\ln(u)]^n - n \int [\ln(u)]^{n-1} du$

**383.**  $\int u \ln(u) du = \frac{u^2}{2} \left[ \ln(u) - \frac{1}{2} \right] + C$

**384.**  $\int u^m \ln(u) du = \frac{u^{m+1}}{m+1} \left( \ln(u) - \frac{1}{m+1} \right) + C$

**385.**  $\int \frac{\ln(u)}{u} du = \frac{1}{2} \ln^2(u) + C$

**386.**  $\int \frac{\ln(u)}{u^2} du = -\frac{\ln(u)}{u} - \frac{1}{u} + C$

**387.**  $\int \ln^2(u) du = u \ln^2(u) - 2u \ln(u) + 2u + C$

**388.**  $\int \frac{\ln^n(u)}{u} du = \frac{\ln^{n+1}(u)}{n+1} + C$

**389.**  $\int \frac{du}{u \ln(u)} = \ln(\ln(u)) + C$

**390.**  $\int \ln(u^2 + a^2) du = u \ln(u^2 + a^2) - 2u + 2a \tan^{-1}\left(\frac{u}{a}\right) + C$

**391.**  $\int \ln(u^2 - a^2) du = u \ln(u^2 - a^2) - 2u + a \ln\left(\frac{u+a}{u-a}\right) + C$

### Fórmulas con $\operatorname{senh}(au)$

**392.**  $\int \operatorname{senh}(au) du = \frac{1}{a} \cosh(au) + C$

**393.**  $\int u \operatorname{senh}(au) du = \frac{u \cosh(au)}{a} - \frac{\operatorname{senh}(au)}{a^2} + C$

**394.**  $\int u^2 \operatorname{senh}(au) du = \left(\frac{u^2}{a} + \frac{2}{a^3}\right) \cosh(au) - \frac{2u}{a^2} \operatorname{senh}(au) + C$

**395.**  $\int \frac{\operatorname{senh}(au)}{u} du = au + \frac{(au)^3}{3 \cdot 3!} + \frac{(au)^5}{5 \cdot 5!} + \dots$

**396.**  $\int \frac{\operatorname{senh}(au)}{u^2} du = -\frac{\operatorname{senh}(au)}{u} + a \int \cosh(au) du$

**397.**  $\int \frac{du}{\operatorname{senh}(au)} = \frac{1}{a} \ln\left[\tanh\left(\frac{au}{2}\right)\right] + C$

**398.**  $\int \operatorname{senh}^2(au) du = \frac{\operatorname{senh}(au) \cosh(au)}{2a} - \frac{u}{2} + C$

**399.**  $\int u \operatorname{senh}^2(au) du = \frac{u \operatorname{senh}(2au)}{4a} - \frac{\cosh(2au)}{8a^2} - \frac{u^2}{4} + C$

**400.**  $\int \frac{du}{\operatorname{senh}^2(au)} = -\frac{\coth(au)}{a} + C$

**401.**  $\int \operatorname{senh}(pu) \operatorname{senh}(qu) du = \frac{\operatorname{senh}[(p+q)u]}{2(p+q)} - \frac{\operatorname{senh}[(p-q)u]}{2(p-q)} + C$

**402.**  $\int u^m \operatorname{senh}(au) du = \frac{u^m \cosh(au)}{a} - \frac{m}{a} \int u^{m-1} \cosh(au) du$

**403.**  $\int \operatorname{senh}^n(au) du = \frac{\operatorname{senh}^{n-1}(au) \cosh(au)}{an} - \frac{n-1}{n} \int \operatorname{senh}^{n-2}(au) du$

**404.**  $\int \frac{\operatorname{senh}(au)}{u^n} du = -\frac{\operatorname{senh}(au)}{(n-1)u^{n-1}} + \frac{a}{n-1} \int \frac{\cosh(au)}{u^{n-1}} du$

**405.**  $\int \frac{du}{\operatorname{senh}^n(au)} = -\frac{\cosh(au)}{a(n-1)\operatorname{senh}^{n-1}(au)} - \frac{n-2}{n-1} \int \frac{du}{\operatorname{senh}^{n-2}(au)}$

### Fórmulas con $\cosh(au)$

**406.**  $\int \cosh(au) du = \frac{\operatorname{senh}(au)}{a} + C$

**407.**  $\int u \cosh(au) du = \frac{u \operatorname{senh}(au)}{a} - \frac{\cosh(au)}{a^2} + C$

**408.**  $\int u^2 \cosh(au) du = -\frac{2u \cosh(au)}{a^2} + \left(\frac{u^2}{a} + \frac{2}{a^3}\right) \operatorname{senh}(au) + C$

**409.**  $\int \frac{\cosh(au)}{u} du = \ln(u) + \frac{(au)^2}{2 \cdot 2!} + \frac{(au)^4}{4 \cdot 4!} + \frac{(au)^6}{6 \cdot 6!} + \dots$

**410.**  $\int \frac{\cosh(au)}{u^2} du = -\frac{\cosh(au)}{u} + a \int \frac{\operatorname{senh}(au)}{u} du$

**411.**  $\int \frac{du}{\cosh(au)} = \frac{2}{a} \tan^{-1}(e^{au}) + C$

**412.**  $\int \cosh^2(au) du = \frac{u}{2} + \frac{\operatorname{senh}(au) \cosh(au)}{2a} + C$

**413.**  $\int u \cosh^2(au) du = \frac{u^2}{4} + \frac{u \operatorname{senh}(2au)}{4a} - \frac{\cosh(2au)}{8a^2} + C$

**414.**  $\int \frac{du}{\cosh^2(au)} = \frac{\tanh(au)}{a} + C$

**415.**  $\int \cosh(qu) \cosh(qu) du = \frac{\operatorname{senh}[(p-q)u]}{2(p-q)} + \frac{\operatorname{senh}[(p+q)u]}{2(p+q)} + C$

**416.**  $\int u^m \cosh(au) du = \frac{u^m \operatorname{senh}(au)}{a} - \frac{m}{a} \int u^{m-1} \operatorname{senh}(au) du$

**417.**  $\int \cosh^n(au) du = \frac{\cosh^{n-1}(au) \operatorname{senh}(au)}{an} + \frac{n-1}{n} \int \cosh^{n-2}(au) du$

**418.**  $\int \frac{\cosh(au)}{u^n} du = -\frac{\cosh(au)}{(n-1)u^{n-1}} + \frac{a}{n-1} \int \frac{\operatorname{senh}(au)}{u^{n-1}} du$

**419.**  $\int \frac{du}{\cosh^n(au)} = \frac{\operatorname{senh}(au)}{a(n-1)\cosh^{n-1}(au)} + \frac{n-2}{n-1} \int \frac{du}{\cosh^{n-2}(au)}$



## ➡ Formulario B: Derivadas

En este formulario:  $c$  es una constante real,  $f, g$  y  $u$  son funciones derivables en  $x$ .

### FÓRMULAS GENERALES

1.  $\frac{d}{dx}(c) = 0$
2.  $\frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x))$
3.  $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
4.  $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
5.  $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
6.  $\frac{d}{dx}[f(u)] = f'(u) \frac{du}{dx}$
7.  $\frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$

### FUNCIONES TRIGONOMÉTRICAS

8.  $\frac{d}{dx}(\sin x) = \cos x$
9.  $\frac{d}{dx}(\cos x) = -\sin x$
10.  $\frac{d}{dx}(\tan x) = \sec^2 x$
11.  $\frac{d}{dx}(\cot x) = -\csc^2 x$
12.  $\frac{d}{dx}(\sec x) = \sec x \tan x$
13.  $\frac{d}{dx}(\csc x) = -\csc x \cot x$
14.  $\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$
15.  $\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$
16.  $\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$
17.  $\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$
18.  $\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$
19.  $\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$

### FUNCIONES LOGARÍTMICAS

20.  $\frac{d}{dx}(e^x) = e^x$
21.  $\frac{d}{dx}(a^x) = a^x \ln a$
22.  $\frac{d}{dx}(\ln x) = \frac{1}{x}$
23.  $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$
24.  $\frac{d}{dx}(e^x) = e^x$
25.  $\frac{d}{dx}(a^x) = a^x \ln a$
26.  $\frac{d}{dx}(\ln x) = \frac{1}{x}$
27.  $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

### FUNCIONES TRIGONOMÉTRICAS INVERSAS

28.  $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
29.  $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
30.  $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
31.  $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$
32.  $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

33. 
$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

34. 
$$\frac{d}{dx}(\operatorname{sen}^{-1} u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

35. 
$$\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

36. 
$$\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \frac{du}{dx}$$

37. 
$$\frac{d}{dx}(\cot^{-1} u) = -\frac{1}{1+u^2} \frac{du}{dx}$$

38. 
$$\frac{d}{dx}(\sec^{-1} u) = \frac{1}{u\sqrt{u^2-1}} \frac{du}{dx}$$

39. 
$$\frac{d}{dx}(\csc^{-1} u) = -\frac{1}{u\sqrt{u^2-1}} \frac{du}{dx}$$

## FUNCIONES HIPERBÓLICAS

40. 
$$\frac{d}{dx}(\operatorname{senh} x) = \cosh x$$

41. 
$$\frac{d}{dx}(\cosh x) = \operatorname{senh} x$$

42. 
$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

43. 
$$\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$$

44. 
$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{senh} x \operatorname{tanh} x$$

45. 
$$\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$$

46. 
$$\frac{d}{dx}(\operatorname{senh} u) = \cosh u \frac{du}{dx}$$

47. 
$$\frac{d}{dx}(\cosh u) = \operatorname{senh} u \frac{du}{dx}$$

48. 
$$\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \frac{du}{dx}$$

49. 
$$\frac{d}{dx}(\coth u) = -\operatorname{cosech}^2 u \frac{du}{dx}$$

50. 
$$\frac{d}{dx}(\operatorname{sec h} u) = -\operatorname{sec h} u \operatorname{tanh} u \frac{du}{dx}$$

51. 
$$\frac{d}{dx}(\operatorname{cosech} u) = -\operatorname{cosech} u \coth u \frac{du}{dx}$$

## FUNCIONES HIPERBÓLICAS INVERSAS

52. 
$$\frac{d}{dx}(\operatorname{senh}^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$

53. 
$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

54. 
$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$$

55. 
$$\frac{d}{dx}(\coth^{-1} x) = -\frac{1}{1-x^2}$$

56. 
$$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$$

57. 
$$\frac{d}{dx}(\operatorname{cosech}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

58. 
$$\frac{d}{dx}(\operatorname{senh}^{-1} u) = \frac{1}{\sqrt{1+u^2}} \frac{du}{dx}$$

59. 
$$\frac{d}{dx}(\cosh^{-1} u) = \frac{1}{\sqrt{u^2-1}} \frac{du}{dx}$$

60. 
$$\frac{d}{dx}(\tanh^{-1} u) = \frac{1}{1-u^2} \frac{du}{dx}$$

61. 
$$\frac{d}{dx}(\coth^{-1} u) = -\frac{1}{1-u^2} \frac{du}{dx}$$

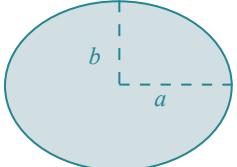
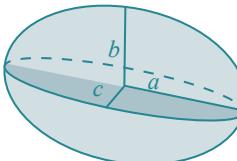
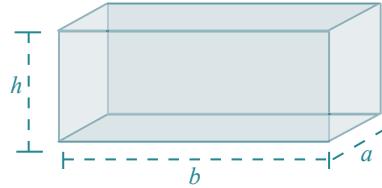
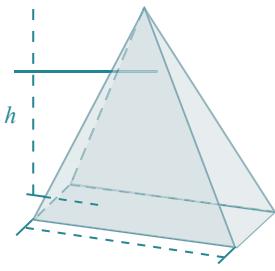
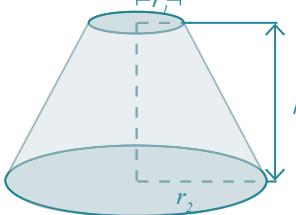
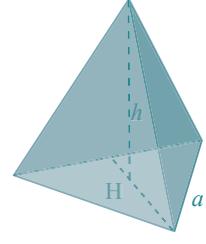
62. 
$$\frac{d}{dx}(\operatorname{sech}^{-1} u) = -\frac{1}{u\sqrt{1-u^2}} \frac{du}{dx}$$

63. 
$$\frac{d}{dx}(\operatorname{cosech}^{-1} u) = -\frac{1}{|u|\sqrt{u^2+1}} \frac{du}{dx}$$

# Formulario C: Álgebra, Geometría y Trigonometría

## Figuras geométricas

<p>Triángulo rectángulo</p> $A = \frac{1}{2}ch = \frac{1}{2}ab, \quad P = a + b + c, \quad c^2 = a^2 + b^2$	<p>Triángulo equilátero</p> $h = \frac{\sqrt{3}}{2}a, \quad A = \frac{\sqrt{3}}{4}a^2, \quad P = 3a$	<p>Cuadrado</p> $A = a^2, \quad P = 4a$
<p>Rectángulo</p> $P = 2b + 2h, \quad A = bh$	<p>Romboide</p> $A = bh$	<p>Trapezoide</p> $A = \frac{1}{2}(a+b)h$
<p>Círculo</p> $A = \pi r^2, \quad P = 2\pi r$	<p>Corona circular</p> $A = \pi(R^2 - r^2), \quad P = 3a$	<p>Sector circular</p> $A = \frac{1}{2}r^2\theta, \quad s = r\theta$
<p>Esfera</p> $V = \frac{4}{3}\pi r^3, \quad S = 4\pi r^2$	<p>Cono circular recto</p> $V = \frac{1}{3}\pi r^2 h, \quad S = \pi r\sqrt{r^2 + h^2}$	<p>Cilindro circular recto</p> $V = \pi r^2 h, \quad S_{\text{lateral}} = 2\pi r h, \quad S_{\text{total}} = 2\pi r h + 2\pi r^2$

<b>Elipse</b>  $A = \pi ab$	<b>Elipsoide</b>  $A = \frac{4}{3}\pi abc$	<b>Paralelepípedo rectangular</b>  $V = abh, S = 2(ab + ah + bh)$
<b>Pirámide</b>  $V = \frac{1}{3}abh$	<b>Cono truncado</b>  $V = \frac{1}{3}\pi h(r_1^2 + 2r_1 r_2 + r_2^2)$	<b>Pirámide Regular</b>  $V = \frac{aH}{2} \left( \frac{1}{3}h \right)$

## → Álgebra

### Fórmula cuadrática

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminante  $b^2 - 4ac$

### Desarrollo de productos notables y factorización

$$(x \pm y)^2 = x^2 \pm 2xy + y^2$$

$$(x \pm y)^3 = x^3 \pm 3(x)^2(y) + 3(x)(y)^2 \pm y^3$$

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \dots + \binom{n}{n-1}x^1y^{n-1} + \binom{n}{n}y^n \quad \forall n \in \mathbb{N}$$

$$\text{Donde } \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$(x^2 - y^2) = (x+y)(x-y)$$

$$(x^3 \pm y^3) = (x \pm y)(x^2 \mp xy + y^2)$$

## Reglas de exponentes y radicales

$$\begin{aligned}x^m x^n &= x^{m+n} & \frac{1}{x^n} &= \sqrt[n]{x} \\(x^m)^n &= x^{m \cdot n} & x^{\frac{n}{m}} &= \sqrt[m]{x^n} \\\frac{x^m}{x^n} &= x^{m-n} & x^{\frac{n}{m}} &= \left(\sqrt[m]{x}\right)^n \\x^{-m} &= \frac{1}{x^m} & \sqrt[n]{(xy)} &= \sqrt[n]{x} \sqrt[n]{y} \\\left(\frac{x}{y}\right)^n &= \frac{x^n}{y^n} & \sqrt[n]{\frac{x}{y}} &= \frac{\sqrt[n]{x}}{\sqrt[n]{y}} \\(xy)^n &= x^n y^n & \sqrt[m]{\sqrt[n]{x}} &= \sqrt[mn]{x}\end{aligned}$$

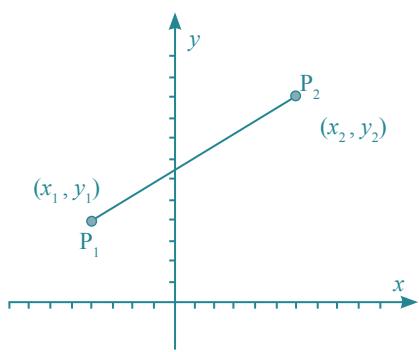
## Valores de exponentiales, propiedades de los logaritmos

$$\begin{aligned}e^{-n} &= \frac{1}{e^n} & e^0 &= 1 \\e^{-\infty} &\rightarrow 0 & e^{\infty} &\rightarrow \infty \\e^{\ln(x)} &= x & a^{\log_a x} &= x \\\log_a(x) &= \frac{\log_{10}(x)}{\log_{10}(a)} & \ln(x) &= \log_e x \\&& \ln(x) + \ln(y) &= \ln(xy) \\&& \ln(x) - \ln(y) &= \ln\left(\frac{x}{y}\right) \\&& n \ln(x) &= \ln(x^n) \\&& \log_a(x) + \log_a(y) &= \log_a(xy) \\&& \log_a(x) - \log_a(y) &= \log_a\left(\frac{x}{y}\right) \\&& b \log_a(x) &= \log_a(x^b)\end{aligned}$$

## Geometría analítica

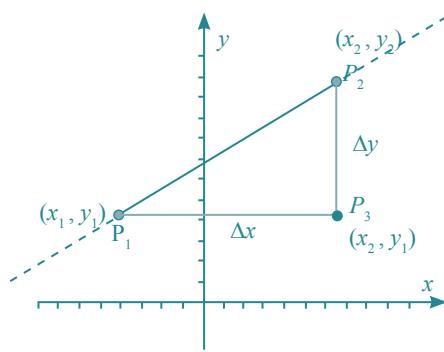
### Distancia entre dos puntos

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



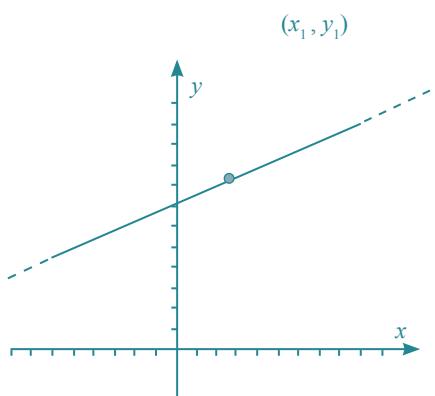
### Pendiente de una recta

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

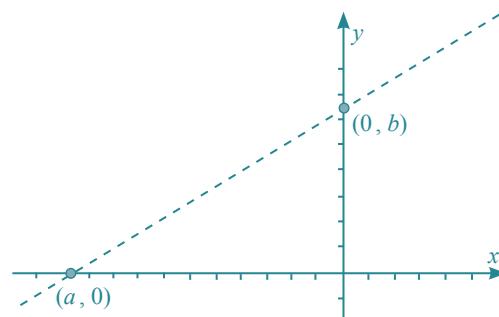


**Ecuación de la recta punto-pendiente**

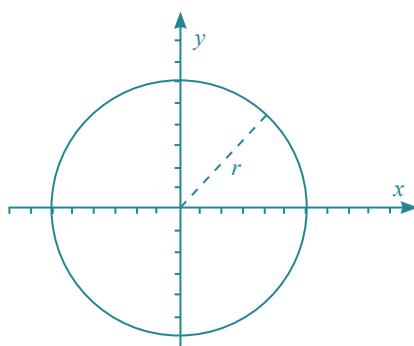
$$y - y_1 = m(x - x_1)$$

**Puntos de intersección de la recta**

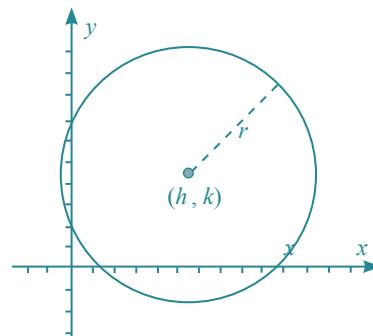
$$\frac{x}{a} + \frac{y}{b} = 1 \quad \forall a \neq 0; b \neq 0$$

**Ecuación de la circunferencia con centro en el origen**

$$x^2 + y^2 = r^2$$

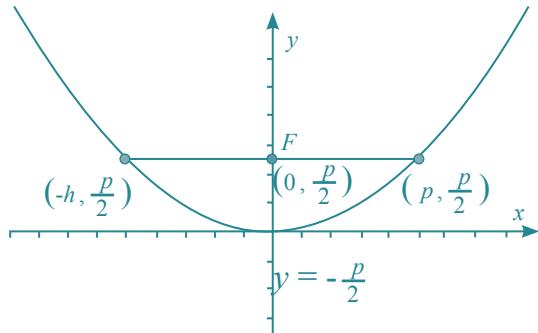
**Ecuación de la circunferencia con centro fuera del origen.**

$$(x - h)^2 + (y - k)^2 = r^2$$

**Parábola**

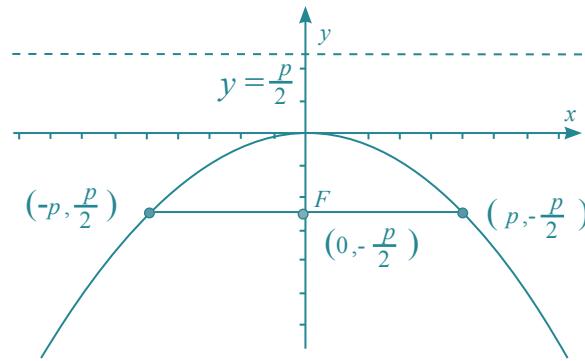
$$x^2 = 2py; \text{ Foco } F = \left(0, \frac{p}{2}\right); \text{ Extremos Izq} \left(-p, \frac{p}{2}\right) \text{ Der} \left(p, \frac{p}{2}\right)$$

$$\text{Lado Recto } Lr = 2p; \text{ Recta Directriz } y = -\frac{p}{2}$$

**Parábola**

$$x^2 = -2py; \text{ Foco } F = \left(0, -\frac{p}{2}\right); \text{ Extremos Izq} \left(-p, -\frac{p}{2}\right) \text{ Der} \left(p, -\frac{p}{2}\right)$$

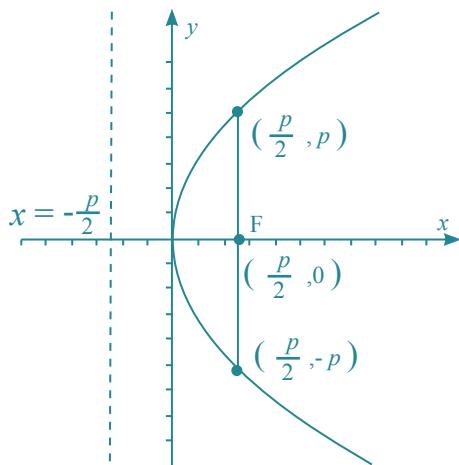
$$\text{Lado Recto } Lr = 2p; \text{ Recta Directriz } y = \frac{p}{2}$$



**Parábola**

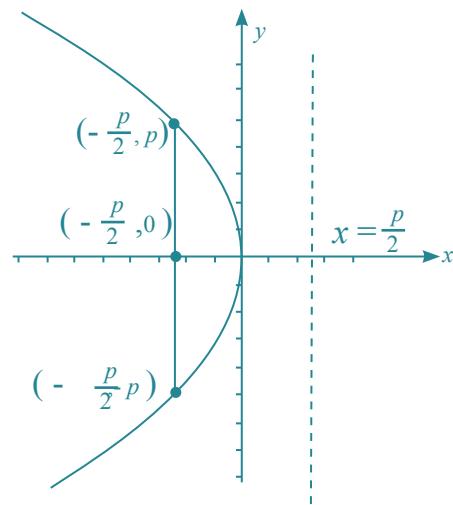
$$y^2 = 2px; \text{ Foco } F = \left(\frac{p}{2}, 0\right); \text{ Extremos Inf } \left(\frac{p}{2}, -p\right); \text{ Sup } \left(\frac{p}{2}, p\right)$$

Lado Recto Lr =  $2p$ ; Recta Directriz  $x = -\frac{p}{2}$

**Parábola**

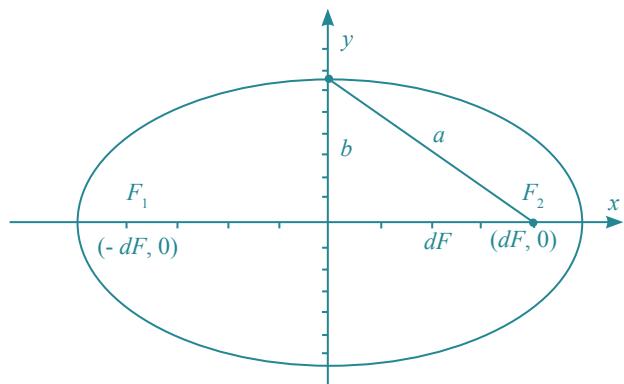
$$y^2 = -2px; \text{ Foco } F = \left(-\frac{p}{2}, 0\right); \text{ Extremos Inf } \left(-\frac{p}{2}, -p\right); \text{ Sup } \left(-\frac{p}{2}, p\right)$$

Lado Recto Lr =  $2p$ ; Recta Directriz  $x = \frac{p}{2}$

**Elipse centro en el origen**

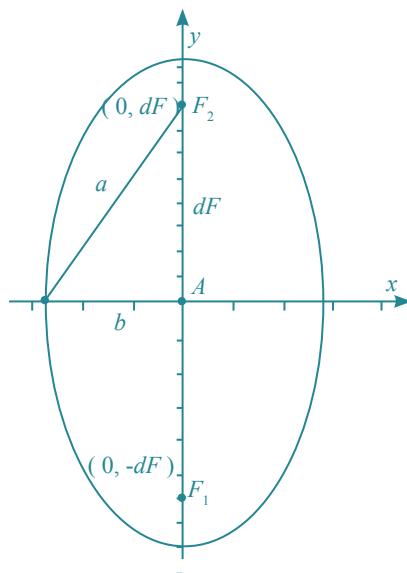
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad dF = \sqrt{a^2 - b^2}$$

$F_1(-dF, 0); F_2(dF, 0)$

**Elipse centro en el origen**

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \quad dF = \sqrt{a^2 - b^2}$$

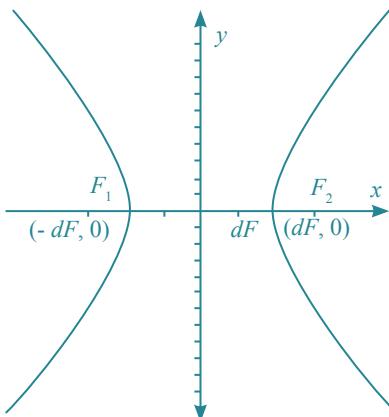
$F_1(0, -dF); F_2(0, dF)$



**Hipérbola**

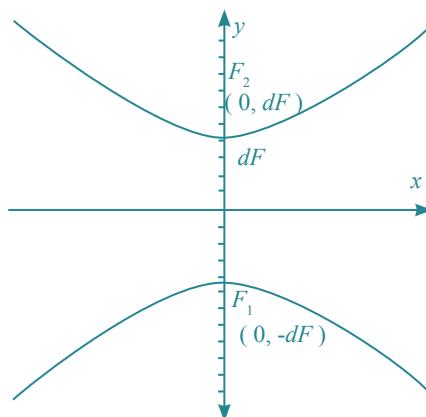
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad dF = \sqrt{a^2 + b^2}$$

$$F_1(-dF, 0); F_2(dF, 0)$$

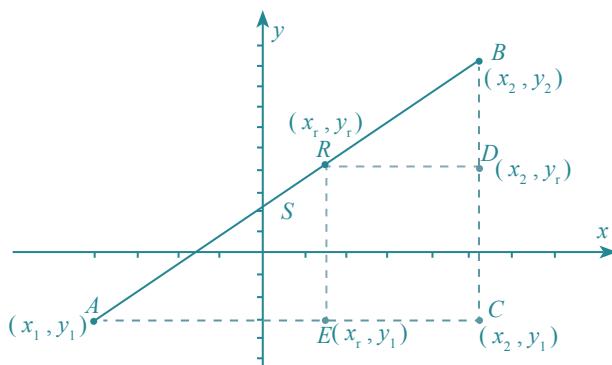
**Hipérbola**

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad dF = \sqrt{a^2 + b^2}$$

$$F_1(0, -dF); F_2(0, dF)$$

**División de un segmento en una razón**

$$r = \frac{x_2 - x_1}{x_r - x_1} \quad r = \frac{y_2 - y_1}{y_r - y_1}$$

**Distancia de un punto a una recta**

Ecuación general de la recta  $Ax + By + C = 0$

$$P(x_1, y_1) \quad \text{Entonces: } d_{Pr} = \left| \frac{Ax_1 + Bx_2 + C}{\sqrt{A^2 + B^2}} \right|$$

